PHYS5702 Quantum Mechanics II Fall 2018 HW #5

Due at 5pm to the **Instructor** on Thursday 15 Nov 2018

(1) Liquid helium undergoes a phase transition to a macroscopic "superfluid" quantum state when the temperature drops below $T = 2.7\,\mathrm{K}$. (For an excellent old movie demonstration, see http://www.alfredleitner.com/p/liquid-helium.html.) Calculate the deBroglie wavelength of helium at that temperature, and use the result to give a physical explanation of why this occurs. Why is it not possible to create a superfluid of neon, the next lightest noble gas?

(2) Two identical spin-1/2 fermions move in one dimension, confined to an infinite square well in the region $0 \leq x \leq L$.

- **(a)** Find the ground state wave function and the ground state energy when the two fermions are in the *singlet* spin state.
- **(b)** Find the ground state wave function and the ground state energy when the two fermions are in the *triplet* spin state.
- **(c)** Find the ground state energy shift from an interaction $V(x_1, x_2) = -\lambda\delta(x_1 - x_2)$, treated as a perturbation, for case **(a)** and case **(b)**.

(3) A porphyrin ring is a molecule which is present in chlorophyll, hemoglobin, and other biological compounds. It can be modeled as 18 electrons moving freely along a one-dimensional circular path of radius $R = 0.4\,\mathrm{nm}$.

- **(a)** Using a polar angular coordinate θ, write down the appropriately normalized single-particle wave functions $\psi(\theta)$, including periodic boundary conditions. Find an expression for the single-particle energy eigenvalues.
- **(b)** Find the electron configurations and energies for the ground state and first excited state of porphyrin.
- **(c)** Find a numerical value for the wavelength of electromagnetic radiation that would excited the ground state into the first excited state. This is a very simple model, and porphyrin comes in many varieties, but compare your result to an experimental result.

(4) A Hamiltonian for a system of bosons has the form

$$\mathcal{H} = \sum_{\mathbf{k}} T(\mathbf{k}) a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \lambda \sum_{\mathbf{l}} \sum_{\mathbf{m}} a^\dagger_{\mathbf{l}} a^\dagger_{-\mathbf{l}} a_{\mathbf{m}} a_{-\mathbf{m}} V(\mathbf{l} + \mathbf{m})$$

Prove that the number operator

$$\mathcal{N} = \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$$

is a constant of the motion.