PHYS5702 Quantum Mechanics II Fall 2018 HW #4

Due at 5pm to the **Instructor** on Thursday 1 Nov 2018

(1) Consider scattering in one dimension x from a potential $V(x)$ localized near $x = 0$. The initial state is a plane wave coming from the left, that is $\phi(x) \equiv \langle x|i\rangle = e^{ikx}/\sqrt{2\pi}$

(a) Find the scattering Green's function $G(x, x')$, defined in one dimension analogously with equation (6.2.3), for $G_+(\mathbf{x}, \mathbf{x}')$.

(b) For the case of an attractive δ-function potential $V(x) = -\gamma\hbar^2\delta(x)/2m$, with $\gamma > 0$, use the Lippman-Schwinger Equation to find the outgoing wave function $\psi(x) \equiv \langle x|\psi^{(+)}\rangle$.

(c) Determine the transmission and reflection coefficients $T(k)$ and $R(k)$, defined as

$$\psi(x) = T(k)\phi(x) \quad \text{for} \quad x > 0 \qquad \text{and} \qquad \psi(x) = \phi(x) + R(k)\frac{e^{-ikx}}{\sqrt{2\pi}} \quad \text{for} \quad x < 0$$

Show that $|T|^2 + |R|^2 = 1$, as must be the case.

(d) Confirm that you get the same result by using grade-school quantum mechanics, matching right and left going waves on the left with a right going wave on the right at $x = 0$.

(e) We showed last semester that this potential has one, and only one, bound state. Show that your results for $T(k)$ and $R(k)$ have bound state poles at the expected positions when k is treated as a complex variable.

(2) An experiment is done by scattering positrons from hydrogen atoms in their ground state. Calculate the elastic scattering differential cross section in the Born Approximation, and find the form factor $F(q)$ where q is the momentum transfer to the atom.

(3) Look up the paper "Negative pion-nucleus elastic scattering at 20 and 40 MeV", by G. Burleson, et al., Phys. Rev. C 49(1994)2226. Use that data to estimate the radius of the ^{40}Ca nucleus, and compare this to the radius deduced from the data shown in Figure 6.6 in your textbook. Comment on the level of agreement or disagreement.

(4) Make use of the eikonal approximation abd $\delta_l = \Delta(b)|_{b=l/k}$ to obtain the phase shift δ_l for scattering at high energies by (a) the Gaussian potential, $V = V_0 \exp(-r^2/a^2)$, and (b) the Yukawa potential, $V = V_0 \exp(-\mu r)/\mu r$. Verify the assertion that δ_l goes to zero very rapidly with increasing l (k fixed) for $l \gg kR$, where R is the "range" of the potential.

(5) Reproduce Figure 6.15 in the textbook, but first realize that there are several errors in the figure and in the accompanying test. The square well parameters satisfy $2mV_0R^2/\hbar^2 = 5.5^2$, not "$= 5.5$" as given in the caption, and the resonance is at $k = 1.41/R$, instead of the $1.3/R$ given in the text. Also, the shapes of the curves as drawn are not accurate, and the vertical axis should be labeled $\delta_3(k)$ for Fig.6.15(b), not $\sigma_3(k)$.