PHYS5702 Quantum Mechanics II Fall 2018 HW #3

Due at 5pm to the **Grader** on Thursday 4 Oct 2018

(1) For $t > 0$, the Hamiltonian for a system of two spin-$\frac{1}{2}$ objects is given by

$$H = \left(\frac{4\Delta}{\hbar^2}\right)\mathbf{S}_1 \cdot \mathbf{S}_2.$$

Suppose the system is in $|+-\rangle$ for $t \le 0$. Find, as a function of time, the probability for being found in each of the following states $|++\rangle$, $|+-\rangle$, $|-+\rangle$, and $|--\rangle$:

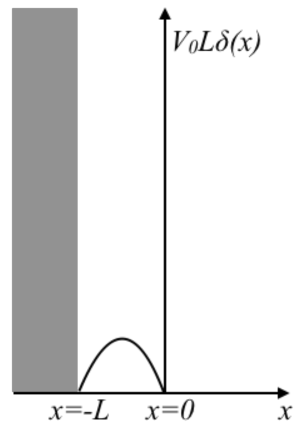
(a) By solving the problem exactly.

(b) By solving the problem using first-order time-dependent perturbation theory. Under what condition does **(b)** give the correct results?

(2) A particle of mass m that moves in one dimension x. There is an infinite wall at $x = -L$, and a δ-function potential $V_0 L\delta(x)$ at $x = 0$. The particle is otherwise free. The (normalized) initial state wave function is given by the ground state of the infinite well bounded by $-L \le x \le 0$, that is

$$\psi(x, t = 0) = -\left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{\pi x}{L}\right)$$

as shown on the right. Find the probability as a function of time that the particle is in the region $-L \le x \le 0$. Under what conditions does this look like "exponential decay"?



(3) The nucleus of a neutral tritium atom undergoes the decay ^{3}H$\to^3$He$+e^- + \bar{\nu}_e$, leaving a singly ionized helium atom. About 18 keV of energy is shared by the electron and neutrino. Identify the appropriate time scales and argue that the sudden approximation is valid to calculate the probability that the helium ion is in its electronic ground and first excited states. Carry out this calculation.

(4) A particle of mass m is in the ground state a finite well in one dimension x, that is the potential is $-V_0$ for $-a \le x \le +a$ and zero otherwise. A perturbation $V(t) = ve^{-x^2/\alpha^2}\sin\omega t$ acts on the particle, where v, α, and ω are constants, and $\alpha \ll a$. Find the transition rate to a state of positive energy E_k. (You can use the solution for a finite well from last semester.)

(5) Find an expression for the decay rate for the $3p \to 2s$ transition in the hydrogen atom. You can find the experimental measurement in "Mean Lives of the $2p$ and $3p$ Levels in Atomic Hydrogen", W. S. Bikel and A. S. Goodman, Phys.Rev. 148(1966)1.