PHYS5702 Quantum Mechanics II Fall 2018 HW #2

Due at 5pm to the **Grader** on Thursday 20 Sep 2018

(1) Calculate the effect of the proton's finite size on the $n = 1$ and $n = 2$ energy levels of the hydrogen atom in perturbation theory. Assume the proton is a uniformly charged sphere of radius R. Explain physically why the $\ell = 1$ shifts are so much smaller than for $\ell = 0$.

(2) In class we derived two of the three relativistic corrections to the one-electron atom, namely $\Delta_K^{(1)}$ from "relativistic kinetic energy," and $\Delta_{LS}^{(1)}$ from the spin-orbit interaction. A third terms comes from the spread of the electron wave function in the region of changing electric field. The perturbation for this "Darwin term" is

$$V_D = -\frac{1}{8m^2c^2}\sum_{i=1}^{3}[p_i, [p_i, e\phi(r)]]$$

where $\phi(r)$ is the Coulomb potential. Find $\Delta_D^{(1)}$ and show that

$$\Delta_{nj}^{(1)} \equiv \Delta_K^{(1)} + \Delta_{LS}^{(1)} + \Delta_D^{(1)} = \frac{mc^2(Z\alpha)^4}{2n^3}\left[\frac{3}{4n} - \frac{1}{j+1/2}\right]$$

Later on in this course we will compare this expression to the result of solving the Dirac equation in the presence of the Coulomb potential.

(3) These questions are meant to associate numbers with atomic hydrogen phenomena.

(a) The red $n = 3 \to 2$ Balmer transition has a wavelength $\lambda \approx 656$ nm. Calculate the wavelength difference $\Delta\lambda$ (in nm) between the $3p_{3/2} \to 2s_{1/2}$ and $3p_{1/2} \to 2s_{1/2}$ transitions due to the spin-orbit interaction. Comment on how you might measure this splitting.

(b) How large an electric field $\mathcal{E}$ is needed so that the Stark splitting in the $n = 2$ level is the same as the correction from relativistic kinetic energy between the $2s$ and $2p$ levels? How easy or difficult is it to achieve an electric field of this magnitude in the laboratory?

(c) The Zeeman effect can be calculated with a "weak" or "strong" magnetic field, depending on the size of the energy shift relative to the spin-orbit splitting. Give examples of a weak and a strong field. How easy or difficult is it to achieve such a magnetic field?

(4) Compute the Stark effect for the $2s_{1/2}$ and $2p_{1/2}$ levels of hydrogen. Assume that the electric field $\mathcal{E}$ is sufficiently weak so that $e\mathcal{E}a_0$ is small compared to the fine structure, but take the Lamb shift δ into account (that is, ignore $2p_{3/2}$ in this calculation). Show that for $e\mathcal{E}a_0 \ll \delta$, the energy shifts are quadratic in $\mathcal{E}$, whereas for $e\mathcal{E}a_0 \gg \delta$ they are linear in $\mathcal{E}$.

(5) Two well separated hydrogen atoms interact via a dipole-dipole interaction of the form

$$V = \frac{e^2}{r^3}\left[\mathbf{r}_1 \cdot \mathbf{r}_2 - 3z_1z_2\right]$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of the two electrons, and r is the distance between the protons. User perturbation theory to find the interaction energy as a function of r. You may assume that all of the energy levels of hydrogen atom are the same distance from the ground state energy. (We haven't yet discussed dealing with "identical" electrons, but make the obvious assumption for the ground state of the two-atom system.)