PHYS5702 Quantum Mechanics II Fall 2018 HW #1

Due at 5pm to the **Grader** on Thursday 6 Sep 2018

(1) A one dimensional potential well has infinite walls at $x = 0$ and $x = L$. The bottom of the well is *not* flat, but rather increases linearly from 0 at $x = 0$ to V at $x = L$. Find the first order shift in the energy levels as a function of principle quantum number n.

(2) A particle of mass m moves in a potential well $V(x) = m\omega^2 x^2/2$. Treating relativistic effects to order $\beta^2 = (p/mc)^2$, find the ground state energy shift.

(3) A diatomic molecule can be modeled as a rigid rotor with moment of inertia I and an electric dipole moment d along the axis of the rotor. The rotor is constrained to rotate in a plane, and a weak uniform electric field $\mathcal{E}$ lies in the plane. Write the classical Hamiltonian for the rotor, and find the unperturbed energy levels by quantizing the angular momentum operator. Then treat the electric field as a perturbation, and find the first non-vanishing corrections to the energy levels.

(4) Consider a Hamiltonian in a two-state system represented by the matrix

$$H = \begin{pmatrix} A & \delta \\ \delta & B \end{pmatrix} \qquad \text{with} \qquad A \leq B$$

(a) Solve exactly for the energy eigenvalues and (matrix representations of) the eigenstates.
(b) Assume $\delta \ll B - A$ and solve with perturbation theory to first order in the eigenstates and second order in the energies. Compare your result to the exact solution.
(c) Assume $A = B$ and apply degenerate perturbation theory to solve the problem. Compare your result to the exact solution for the case $\delta \gg B - A$.

(5) Consider the isotropic harmonic oscillator in two dimensions, with the Hamiltonian

$$H = H_0 + m\omega^2\epsilon\, xy \qquad \text{where} \quad H_0 = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}m\omega^2(x^2 + y^2)$$

Find the energy eigenvalues of the three lowest lying states for H_0, and identify any degeneracy. Then treat the term with ϵ as a perturbation and find the first order energy shift for each of these states. Finally, solve the problem exactly and compare to the solution you obtained with perturbation theory.