PHYS5701 Quantum Mechanics I Spring 2018 HW #6

Due at 5pm to the <u>**Grader**</u> on Thursday 12 Apr 2018

(1) For the orbital angular momentum operator $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, derive (3.6.9), that is

$$\langle \mathbf{x}' | L_z | \alpha \rangle = -i\hbar \frac{\partial}{\partial \phi} \langle \mathbf{x}' | \alpha \rangle$$

by using the standard spherical coordinate transformation

$$x' = r\cos\phi\sin\theta \qquad y' = r\sin\phi\sin\theta \qquad z' = r\cos\theta$$

(2) Assume the wave function $\psi(r,\theta,\phi) = [u(r)/r]\, Y_l^m(\theta,\phi)$ satisfies the Schrödinger Equation for a spherically symmetric potential. Show that, for $r \to 0$, there is a general solution $u(r) = Ar^{l+1} + Br^{-l}$. Then show that $B \neq 0$ leads to an unphysical situation where the origin is a source of probability. *Note that there is an error in the textbook's explanation.*

(3) Consider the energy eigenvalues for a spherically symmetric "box" of radius a.

(a) For the box with infinite walls, check the eigenvalues for the $l = 0$, $l = 1$, and $l = 2$ states, given in (3.7.25), (3.7.26), and (3.7.27). *Beware: There is an error in the book!*

(b) Find the lowest energy eigenvalues with $l = 0$ for a finite spherical box with potential wall height $V_0 = \hbar^2\beta^2/2ma^2$ where $\beta = 4$, 10, 25, and 100, and show that your numerical results approach the appropriate value given in **(a)**.

(4) Consider the Coulomb potential $V(\mathbf{x}) = -Ze^2/r$ (3.7.43) and define the (quantum mechanical operator analog of the) Runge-Lenz vector

$$\mathbf{M} = \frac{1}{2m}(\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}) - \frac{Ze^2}{r}\mathbf{x}$$

(a) Prove that $\mathbf{M}$ is Hermitian.

(b) Prove that $\mathbf{M}$ commutes with the Hamiltonian

We will return to $\mathbf{M}$ when we go through Pauli's algebraic solution for this Hamiltonian.

(5) A spin-1/2 particle is in an orbital angular momentum $l = 1$ state.

(a) Starting with the total angular momentum state $|j, m\rangle = |j, j\rangle$ for $j = 3/2$, use the operator $\mathbf{J} = \mathbf{L} + \mathbf{S}$ to construct all the states $|j, m_j\rangle$ in terms of the eigenstates $|l, m_l\rangle$ for $\mathbf{L}^2$ and L_z, and the eigenstates $|1/2, \pm 1/2\rangle$ for $\mathbf{S}^2$ and S_z. Check your answers against any available table of Clebsch-Gordan coefficients. I like to use this one:

http://pdg.lbl.gov/2017/reviews/rpp2017-rev-clebsch-gordan-coefs.pdf

There is also the MATHEMATICA function ClebschGordan.

(b) If the particle is in a total angular momentum eigenstate with z-component $+\hbar/2$, calculate the probability of finding the z-component of the spin of the particle to have the value $m_s = +1/2$.