PHYS5701 Quantum Mechanics I Spring 2018 HW #5

Due at 5pm to the <u>**Grader**</u> on Thursday 29 Mar 2018

(1) Let $\hat{\mathbf{n}}$ be a unit vector that makes a polar angle θ with respect to the $\hat{\mathbf{z}}$ axis, and azimuthal angle ϕ with respect to the $\hat{\mathbf{x}}$ axis. (That is, θ and ϕ are just the normal physicist standard spherical coordinates.) Find the eigenvalues ϵ and eigenvectors χ

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}\chi = \epsilon\chi$$

where χ is a two-component spinor. Explain the result physically in terms of spin-1/2.

(2) Use the triangle inequality (1.4.54) and the definition (3.4.8) of the density operator ρ to prove that $0 \leq \mathrm{Tr}(\rho^2) \leq 1$.

(3) A large collection of spin-1/2 particles is in a mixture of the two states $|S_z;+\rangle$ and $|S_y;-\rangle$. The fraction of particles in the state $|S_z;+\rangle$ is a. Find the ensemble averages $[S_x]$, $[S_y]$, and $[S_z]$ in terms of a. Confirm that your expression gives the answers you expect for $a = 0$ and $a = 1$.

(4) Construct the matrix representations of the operators J_x and J_y for a spin-one system, in the J_z basis, spanned by the kets $|+\rangle \equiv |1,1\rangle$, $|0\rangle \equiv |1,0\rangle$, and $|-\rangle \equiv |1,-1\rangle$. Use these matrices to find the three analogous eigenstates for each of the two operators J_x and J_y in terms of $|+\rangle$, $|0\rangle$, and $|-\rangle$.

(5) Modern Quantum Mechanics, 2nd Edition, Problem 3.26. It will be helpful to review the text on pages 198 and 199.