PHYS5701 Quantum Mechanics I Spring 2018 HW #4

Due at 5pm to the **Grader** on Thursday 15 Mar 2018

(1) This problem asks you to generalize two familiar one-dimensional eigenvalue and wave function problems to three dimensions. For each of the following, determine the first three energy eigenvalues, and the number of states with each energy (that is, the degeneracies of the energy levels.) Find also the "parity" of the wave functions for each energy level; a wave function has a positive or negative parity according to the transformation $\psi(-\mathbf{x}) = \pm\psi(\mathbf{x})$.

(a) A cubic box of side length a with infinitely high walls, centered at the origin.
(b) An isotropic harmonic oscillator, that is, $V(\mathbf{x}) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$.

(2) A one dimensional simple harmonic oscillator with natural frequency ω is an initial state

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\delta}}{\sqrt{2}}|1\rangle$$

(a) Find the time-dependent wave function $\langle x'|\alpha;t\rangle$ and evaluate the (time-dependent) expectation values $\langle x\rangle$ and $\langle p\rangle$ in the state $|\alpha;t\rangle$, i.e. in the Schrödinger picture.
(b) Now calculate $\langle x\rangle$ and $\langle p\rangle$ in the Heisenberg picture and compare the results.

(3) Use the WKB method to find the (approximate) energy eigenvalues for the one-dimensional simple harmonic oscillator potential $V(x) = m\omega^2 x^2/2$. Does the answer surprise you?

(4) A particle of mass m moves along one of two "paths" through space and time connecting the points $(x,t) = (0,0)$ and $(x,t) = (D,T)$. One path is quadratic in time, i.e. $x_1(t) = \frac{1}{2}at^2$ where a is a constant. The second path is linear in time, i.e. $x_2(t) = vt$ where v is a constant. The correct classical path is the quadratic path, that is $x_1(t)$.

(a) Find the acceleration a for the correct classical path. Use freshman physics to find the force $F = ma = -dV/dx$ and then the potential energy function $V(x)$ in terms of m, D, and T. Also find the velocity v for the linear (i.e. incorrect classical) path.
(b) Calculate the classical action $S[x(t)] = \int_0^T \left[\frac{1}{2}m\dot{x}^2 - V(x)\right] dt$ for each of the two paths $x_1(t)$ and $x_2(t)$. Confirm that $S_1 \equiv S[x_1(t)] < S_2 \equiv S[x_2(t)]$, and find $\Delta S = S_2 - S_1$.
(c) Calculate $\Delta S/\hbar$ for a particle which moves 1 mm in 1 ms for two cases. The particle is a nanoparticle made up of 100 carbon atoms in one case. The other case is an electron. For which of these would you consider the motion "quantum mechanical" and why?

(5) Modern Quantum Mechanics, 2nd Edition, Problem 2.28.