PHYS5701 Quantum Mechanics I Spring 2018 HW #3

Due at 5pm to the **Grader** on Thursday 22 Feb 2018

(1) A particle with mass m moves in one dimension and is acted on by a constant force F. Find the operators $x(t)$ and $p(t)$ in the Heisenberg picture, and find their expectation values for an arbitrary state $|\alpha\rangle$. Use $\langle x(0)\rangle = x_0$ and $\langle p(0)\rangle = p_0$. The result should be obvious. Comment on how to do this problem in the Schrödinger picture, but *don't try to do it out.*

(2) A non-relativistic particle with mass m is subject to a three-dimensional potential energy $V(\mathbf{x})$. Calculate the commutator $[[x_i, H], x_i]$ for $i = 1, 2, 3$. Use this to show that

$$\sum_k |\langle E_k|\mathbf{x}|E_0\rangle|^2(E_k - E_0) = \frac{3\hbar^2}{2m}$$

where the E_k are eigenvalues of the Hamiltonian H. This is knows as the Dipole Sum Rule, a useful tool for evaluating the strength of electromagnetic transitions in atoms and nuclei.

(3) Modern Quantum Mechanics, 2nd Edition, Problem 2.19.

(4) This exercise has to do with proving conservation laws in classical and quantum physics.

- **(a)** Show that if a quantity Q with a density $\rho(\mathbf{x}, t)$ in some region $\mathcal{R}$ can only be changed by a flux density $\mathbf{j}(\mathbf{x}, t)$ through the surface bordering $\mathcal{R}$, then

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{j} + 0$$

- **(b)** Prove that Maxwell's Equations imply that electric charge is conserved
- **(c)** Prove that Schrödinger's Equation implies that probability is conserved, i.e. (2.4.15)

(5) A particle of mass m is confined to a one-dimensional square well with finite walls. That is, a potential $V(x) = 0$ for $-a \le x \le +a$, and $V(x) = V_0 = \eta(\hbar^2/2ma^2)$ otherwise. You are to find the bound state energy eigenvalues as $E = \epsilon V_0$ along with their wave functions.

- **(a)** Set the problem up with a wave function $Ae^{\alpha x}$ for $x \le -a$, $De^{-\alpha x}$ for $x \ge +a$, and $Be^{ikx} + Ce^{-ikx}$ inside the well. Match the boundary conditions at $x = \pm a$ and show that k and α must satisfy $z = \pm z^*$ where $z \equiv e^{iak}(k - i\alpha)$. Proceed to find a purely real or purely imaginary expression for z in terms of k and α.
- **(b)** Find the wave functions for the two choices of z and show that the purely real (imaginary) choice leads to a wave function that is even (odd) under the exchange $x \to -x$.
- **(c)** Find a transcendental equation for each of the two wave functions relating η and ϵ. Show that even a very shallow well ($\eta \to 0$) has at least one solution for the even wave function, but you are not guaranteed any solution for an odd wave function.
- **(d)** For $\eta = 10$, find all the energy eigenvalues and plot their normalized wave functions.