PHYS5701 Quantum Mechanics I Spring 2018 HW #1

Due at 5pm to the **Grader** on Thursday 25 Jan 2018

(1) A beam of silver atoms is created by heating a vapor in an oven to 1000°C, and selecting atoms with a velocity close to the mean of the thermal distribution. The beam moves through a one-meter long magnetic field with a vertical gradient 10 T/m, and impinges a screen one meter downstream of the end of the magnet. Assuming the silver atom has spin-1/2 with a magnetic moment of one Bohr magneton, find the separation distance in mm of the two states on the screen.

(2) An operator in a two-state system is written as

$$X = \alpha|1\rangle\langle 1| + \beta|2\rangle\langle 2| + \gamma|2\rangle\langle 1| + \delta|1\rangle\langle 2|$$

where α, β, γ, and δ are real numbers, and $|1\rangle$ and $|2\rangle$ are orthonormal vectors.

- **(a)** Find the eigenvalues of X.
- **(b)** Find the eigenvectors in terms of $|1\rangle$ and $|2\rangle$. (Don't bother to normalize them.)
- **(c)** Under what condition is X a Hermitian operator? Show that, in this case, the eigenvalues are real.

You may want to use a symbolic manipulation program to carry out that somewhat messy algebra.

(3) For the spin-1/2 state $|S_x;+\rangle$, evaluate both sides of the inequality (1.4.53), that is

$$\left\langle (\Delta A)^2 \right\rangle \left\langle (\Delta B)^2 \right\rangle \geq \frac{1}{4} \left|\langle [A,B] \rangle\right|^2$$

for the operators $A = S_x$ and $B = S_y$, and show that the inequality is satisfied. Repeat for the operators $A = S_z$ and $B = S_y$.

(4) For arbitrary operators X and Y, a Hermitian operator A, and an operator $Z = \alpha|a\rangle\langle b|$,

- **(a)** Prove that the trace of XY equals the trace of YX
- **(b)** Prove that $(XY)^\dagger = Y^\dagger X^\dagger$
- **(c)** Find an expression for $Z^\dagger$
- **(d)** Evaluate $\langle a|\exp[f(A)]|a\rangle$ where $f(x)$ is an arbitrary function and $|a\rangle$ is a normalized eigenvector of A with eigenvalue a.

(5) Given the four 2×2 matrices σ_k,

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

define the matrix $X = \sum_k a_k \sigma_k$ where the a_k are numbers. Then,

- **(a)** Show that $\sigma_k^2 = 1$ for each k
- **(b)** Find the trace of X in terms of the a_k
- **(c)** Find the trace of $\sigma_k X$ in terms of the a_k