DETAILED MEASUREMENTS OF THE QUANTIZED FLUX STATES OF HOLLOW SUPERCONDUCTING CYLINDERS*

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We have measured the magnetic flux trapped in superconducting tin cylinders (56-μm i.d., 0.7- to 5-μm walls, 24 mm long) and find that for most values of applied magnetic field the entire cylinder is in the same quantized flux state trapping an integral multiple of $hc/2e$ ±1%. For some applied fields we have evidence that the cylinder exists in a mixed state with bands along the length in states differing by one flux quantum.

The first experimental demonstration of fluxoid quantization was obtained through measurements of quantized flux trapped in hollow superconducting cylinders.[1-3] London had introduced the concept of the fluxoid Φ defined for any closed path within a superconductor by

$$\Phi = \oint (4\pi\lambda^2/c)\vec{j}\cdot d\vec{s} + \oint \vec{A}\cdot d\vec{s} = n\Phi_0, \quad (1)$$

where λ is the penetration depth, $\vec{j}$ the supercurrent density, and $\vec{A}$ the magnetic vector potential, and he had predicted that it was quantized.[4] The quantum Φ_0 determined experimentally and now related to the pairing of electrons in superconductors by the microscopic theory is

$$\Phi_0 = hc/2e = 2.07\times10^{-7}\ \text{G cm}^2. \quad (2)$$

For a superconducting cylinder with wall thickness large compared with the penetration depth the first integral in (1) can be negligibly small, and the total magnetic flux trapped by a persistent current in the cylinder becomes precisely quantized in units of Φ_0. Recent measurements by Lischke[5] give this result within ±4%.

An important question raised by the first quantized flux measurements is, What occurs in applied fields corresponding to the transition between quantized flux states? For example, does the cylinder sometimes trap no flux and sometimes trap one flux unit so that the average value of the trapped flux for many trials lies somewhere between zero and one? Experiments which measure the average of many trappings do, in fact, show a continuous variation of the total flux with applied field.[6]

The surprising results of our measurements (Fig. 1) are that the trapped flux appears to vary continuously even for individual trappings. This is interpreted by Pierce[7] to mean that for certain applied fields the cylinder is not in a single quantized flux state but that bands along the length exist in different states. With more detailed measurements we find that although for most values of applied field the entire cylinder is in the same quantized flux state (with trapped flux equal to $n\Phi_0 \pm 1\%$), for a certain range of fields there is a nonuniform distribution of flux in the cylinder. We have evidence that bands along the length exist in states differing by one flux quantum. The bands are distributed along the length of the cylinder so that the trapped flux appears to vary almost continuously with applied field when the average flux in the cylinder or total magnetic moment of the cylinder is measured.

We present here the results of measurements we have made on tin cylinders 56 μm i.d., 24 mm long, and with wall thicknesses from 0.14 to 5 μm. Most of the samples were prepared by evaporating tin in a vacuum of approximately 10^{-6} Torr onto an insulated copper wire, and they were enclosed in small glass tubes.

All the measurements were made with a superconducting magnetometer similar in concept to that described by Silver and Zimmerman.[8] A niobium ring with an adjustable point contact was coupled to the inductor of a tank circuit tuned to 20 MHz and driven with a constant current source. A small af magnetic field was

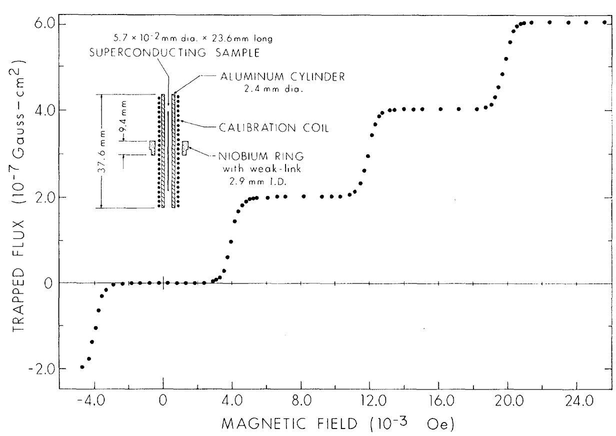

FIG. 1. Trapped flux as a function of the magnetic field in which the cylinder was cooled below the superconducting transition temperature. (Inset) Diagram of magnetometer and sample.

applied to the niobium ring producing a modulation of the rf which was phase sensitive to the magnetic flux linking the ring. This audio signal was fed to a lock-in amplifier whose output was fed back to a long solenoid inside the niobium ring. When the feedback loop was closed the flux within the ring was constrained to remain constant, and the current flowing in the solenoid became a direct measure of flux changes through the ring.

The configuration used for trapped-flux measurements is shown in the inset in Fig. 1. The solenoid marked "calibration coil" was used for the feedback current from the lock-in amplifier. The aluminum cylinder surrounding the sample was many skin depths thick at 20 MHz to shield the sample from rf fields.

Trapped-flux measurements were made by placing the sample in a uniform magnetic field, cooling through the transition temperature, then removing the sample from the field and inserting it into the magnetometer. The sample was then heated above the transition temperature and allowed to cool again; the change in magnetometer reading was a measure of the trapped flux. Some results are shown in Fig. 1. Several conclusions can be drawn from these data: For most values of trapping field the trapped flux is an integral multiple of Φ_0 within $\pm 1\%$. The sizes of the four steps measured are identical within 1%. The product of the incremental magnetic field between two steps and the sample cross-sectional area is equal to one flux quantum.

The measured values of trapped flux lying between integral flux values are repeatable and independent of temperature from about 0.010 K below the transition temperature to approximately 2.0 K. Further, the flux trapped at a given temperature is unchanged if the temperature is subsequently lowered. In cases in which the measured value of the trapped flux fell between integral quantized values the magnetometer output always showed some variation as a function of sample position. However, flux was distributed throughout the full length of the sample but with reduced average value.

Measurements of this type were made on five samples. The data in Fig. 1 are for a sample with walls 0.70 μm thick evaporated onto an insulated copper wire. Essentially identical results were obtained with films evaporated on insulated wires cooled with liquid nitrogen and for a sample with walls 5 μm thick electroplated on copper wire.

A sample with walls 0.14 μm thick of evaporated tin showed the same general shape for the trapped-flux curve, but the value of trapped flux along the steps was 3.5% lower than Φ_0. Since the assumption that the wall thickness is many penetration depths is not valid for this sample, the first integral in (1) should make a contribution and thus a reduced trapped flux is expected.

Although the repeatability of the magnetometer was somewhat better, the absolute value of the flux is known only to $\pm 1\%$. An absolute calibration was obtained from the known geometry of the calibration coil (Fig. 1 inset). Another calibration made use of the fact that when the feedback loop was opened, the output of the magnetometer was a periodic function of the flux through the niobium ring with period Φ_0. The calibrations are consistent to better than 1%. The absolute accuracy of the measurements is limited by the precision with which we know the diameter of the calibration coil.

A second kind of measurement yielded the data shown in Fig. 2. For these measurements a uni-

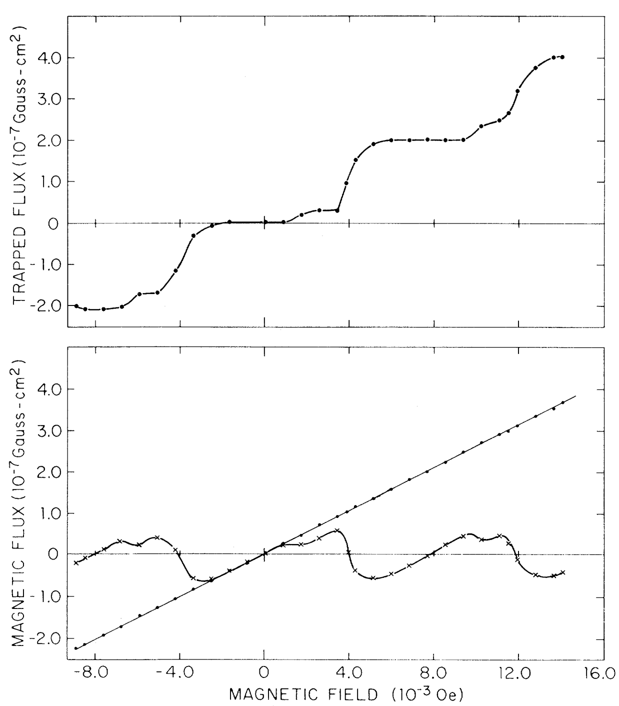

FIG. 2. (Upper) Trapped flux as a function of the magnetic field in which the cylinder was cooled below the superconducting transition temperature. (Lower) Flux produced by the cylinder in the field in which it was cooled below the transition temperature. The straight line shows the flux produced by the cylinder when it is inserted into the applied field already superconducting and with no flux trapped.

form field was applied within the magnetometer ring. The sample was cooled through the transition temperature in zero applied field outside the magnetometer and then inserted into the magnetometer ring. The resulting flux change corresponds to the total expulsion of the applied field from the cross-sectional area of the sample and yields the straight line in Fig. 2. (This provides still another means of calibrating the magnetometer and is consistent with the previously mentioned calibrations.) The sample was then heated and allowed to cool again through the transition temperature in the presence of the applied field. The flux change then observed yields the curve just below the straight line in Fig. 2 and corresponds to the magnetization of the cylinder in the presence of the field. Finally, the applied field was turned off, the sample heated above the transition to release the trapped flux, and the corresponding flux change measured, giving the trapped-flux curve shown in the upper part of Fig. 2. The difference between the two lower curves gives exactly the trapped-flux curve. This indicates that the mixed state of the cylinder is chosen when the cylinder is cooled below the transition temperature in the field and is not changed when the field is turned off.

The points on the trapped-flux curve lying between the flat steps in Fig. 1 imply that the cylinder is not in a single quantized flux state. In order to investigate this possibility and, specifically, prompted by a suggestion of Pierce that bands along the length of the cylinder might exist in different quantum states, we measured the flux as a function of position along the cylinder. This was done by using a pair of superconducting coils connected in a continuous circuit to transfer flux from a small sensing coil into the magnetometer. One coil, 0.04 cm diam and 0.02 cm long, was placed well outside the magnetometer; the second, 0.08 cm diam and 1.56 cm long, was placed inside the aluminum cylinder within the niobium ring. As before, flux was trapped in the cylinder by cooling it through its transition in the presence of an applied field, and then the field was turned off. The sample was then passed slowly through the small loop and the magnetometer output recorded.

We have made many such maps of the flux; some representative results are shown in Fig. 3. Maps corresponding to trapped-flux values lying on the steps in Fig. 1 show a uniform distribution of flux along the entire cylinder. The curves

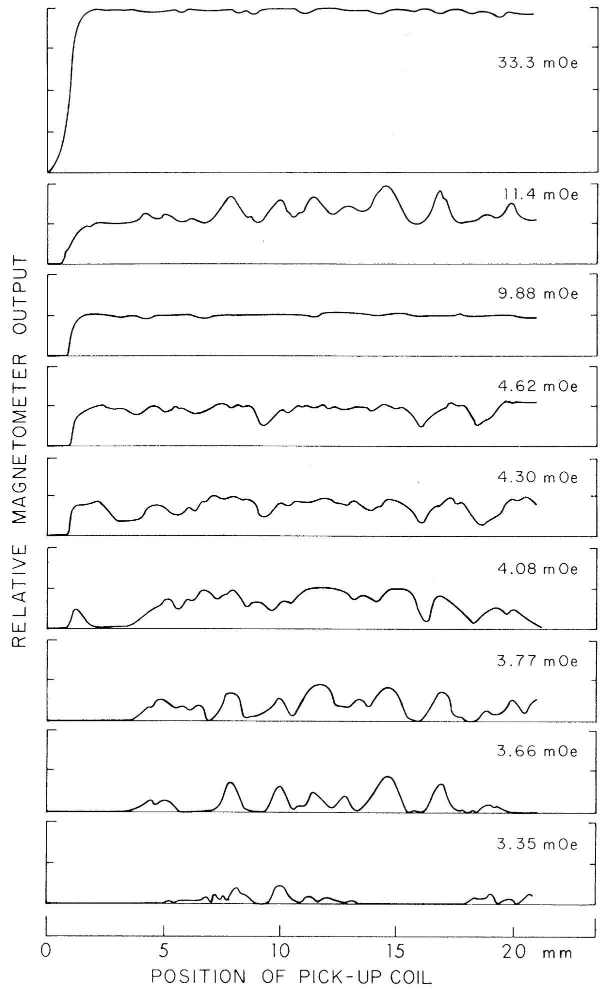

FIG. 3. Magnetometer output as a function of position of the pickup coil along the length of the cylinder. The curves are labeled with the value of the applied field in which the sample was cooled through the transition temperature.

for 9.88 and 33.3 mOe correspond to one and four flux units trapped, respectively. A sequence of maps (3.35-4.62 mOe) is shown corresponding to flux values lying on the curve between zero and the first step in Fig. 1. For this range of trapping fields there are some regions with trapped flux and others with none. Small field changes cause large changes in the flux pattern, e.g., the curves for 3.66 and 3.77 mOe. For fields less than about 4 mOe bands exist either in the zero-flux state or with one flux unit trapped. For fields slightly larger than 4 mOe this pattern is inverted with most of the cylinder having one flux unit trapped but some regions indicating reduced flux. The curve for 11.4 mOe corresponds to a point lying between the first and second steps in Fig. 1. Note that this curve is essentially the sum of the map for 3.66 mOe and that for 9.88 mOe. In general, we find that for trapping fields corresponding to points between the nth and $(n+1)$st steps in Fig. 1, the flux maps indicate regions with no less than n flux units and no more than $n+1$ flux units; further, the average value of the flux over the whole cylinder corresponds to some point lying between the two steps in Fig. 1. Since the larger diameter magnetometer ring used for obtaining the data in Fig. 1 senses an average over a significant fraction of the length of the sample, the correspondence seems good.

It is possible to have the cylinder exhibit more complicated flux patterns by trapping flux, slowly warming the cylinder, then quickly recooling as the flux begins to change. In one map following such a procedure we found regions corresponding to 1, 2, and 3 trapped quanta.

The fact that trapped flux tends to appear in the same geometrical regions on the cylinder repeatedly as the trapping field is raised probably indicates nonuniformities either in the sample or in the local field environment.

In summary then, we find that for most values of applied field the flux is distributed uniformly through the entire cylinder and the values of trapped flux are integral multiples of $(hc/2e)$ $\pm 1\%$. For some values of applied field there is a nonuniform distribution of flux along the length of the cylinder. This feature is interpreted as bands existing in quantized flux states with different quantum numbers along the length of the cylinder and distributed in such a way that average flux in the cylinder varies smoothly with trapping field.[7] Further, we find that this mixed state is established in the presence of the applied field when the sample is cooled below its transition and is unaffected by turning the field off, or by further cooling.

This distribution of bands of different quantum number can be imagined pictorially as a quantized flux line weaving in and out of the wall of the cylinder and being pinned there. The possibility of flux leaking through the wall at a normal spot was cited as a possible explanation for measured flux values lying between quantized steps in the original measurements[1-3] and further evidence has been given in the measurements of Lischke.[4] Observations of flux pinned as localized single flux quanta in solid Nb wires have been reported previously by Zimmerman and Mercereau,[9] who used a mapping technique similar to that used in our measurements.

We want to thank John M. Pierce for suggesting the interpretation of these results and for very many helpful discussions.

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