PHYS5701 Quantum Mechanics I Fall 2020 HW #7

Due at 5pm to the **<u>Instructor</u>** on Tuesday 8 Dec 2020

You are to work independently on this problem set. You are free to use whatever notes, books, computers, or other reference works you feel are useful. You are also free to consult the course instructor or teaching assistant for help. You may *not*, however, consult other students in the class.

Please attach this page to your homework solutions along with your signature, below.

"I have complied with the requirement that I work independently on this problem set. I have not consulted with anyone other than the course instructor and teaching assistant in preparing these solutions."

Signature: ______________________________

Print name: ______________________________

Please note that there are a total of <u>seven</u> problems. All will be equally weighted.

HW #7 (Seven Problems)

(1) MQM3e Problem 1.33

(2) Consider two distinct spin-1/2 particles, labeled "1" and "2". Their spin states are acted on by operators $\mathbf{S}_1$ and $\mathbf{S}_2$, respectively. Their joint basis can be written as $|m_1, m_2\rangle$ where $m_1 = \pm 1/2$ and $m_2 = \pm 1/2$ are the quantum numbers for the S_{1_z} and S_{2_z} operators, respectively. Interpreting the operator $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$ where each term acts on its relevant component in the $|m_1, m_2\rangle$ basis,

(a) Find the matrix representation of the operator $\mathbf{S}^2$ in the $|m_1, m_2\rangle$ basis.

(b) Diagonalize this matrix to find the four eigenvalues of $\mathbf{S}^2$ and the associated eigenvectors.

(c) Show that these eigenvectors are also eigenvectors of the operator $S_z = S_{1_z} + S_{2_z}$ and determine the associated eigenvalues.

(3) MQM3e Problem 3.26

(4) A particle of mass m is confined to a three-dimensional infinite spherical well. The radius of the well is a. Write down, or otherwise determine, the (complete) energy eigenfunction for the lowest $l = 3$ state with (L_z eigenvalue) $m = -1$ and also its energy eigenvalue.

(5) Using some computer application, carry through explicit calculations of the expectation values for the operators $\mathbf{x}^2$ and z^2 in the $|nlm\rangle = |321\rangle$ state of a one-electron atom with atomic number Z. Express your answers in terms of the Bohr radius a_0.

(6) Many of the so-called "elementary" particles possess an internal symmetry called "isotopic spin" or "isospin". The generator of rotations in "isospin space" is called $\mathbf{T}$, and is completely analogous to the angular momentum operator $\mathbf{J}$. For example, the proton p and neutron n are both manifestations of a single particle (the "nucleon" N), which has isospin $t = 1/2$, where $|p\rangle = |t = 1/2, t_z = 1/2\rangle$ and $|n\rangle = |t = 1/2, t_z = -1/2\rangle$. Another example is the π meson having $t = 1$, which exists in three varieties, namely π^+, π^0, and π^-, corresponding to $t_z = +1$, 0, and -1 respectively. A third example is the Δ particle, with $t = 3/2$, existing as Δ^{++}, Δ^+, Δ^0, and Δ^-, that is $t_z = +3/2$, $+1/2$, $-1/2$, and $-3/2$ respectively. The Δ is short lived, and decays to a nucleon and a pion, that is, $\Delta \to N\pi$.

Assuming isospin is a conserved quantity in all relevant interactions concerning Δ decay, calculate the decay probability for $\Delta^0 \to n\pi^0$ relative to the probability for $\Delta^0 \to p\pi^-$.

(7) MQM3e Problem 3.45