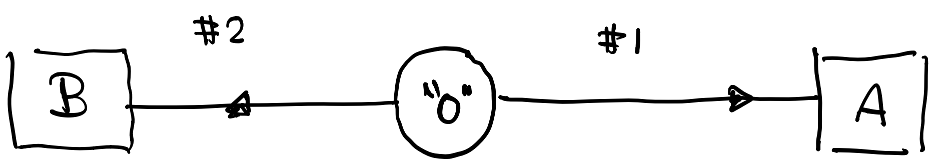
Phys 5701 1 Dec 2020

$$|\text{"spin singlet", i.e. spin-0}\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\,|\underset{\#1}{\hat{z}+};\underset{\#2}{\hat{z}-}\rangle - |\hat{z}-;\hat{z}+\rangle\,\right] \Leftarrow$$



(1.110a) in a different notation:

$$|\hat{x}+\rangle = \frac{1}{\sqrt{2}}\left[\,|\hat{z}+\rangle + |\hat{z}-\rangle\,\right]$$

$$|\hat{x}-\rangle = \frac{1}{\sqrt{2}}\left[\,|\hat{z}+\rangle - |\hat{z}-\rangle\,\right]$$

$$\Leftrightarrow \quad |\hat{z}\pm\rangle = \frac{1}{\sqrt{2}}\left[\,|\hat{x}+\rangle \pm |\hat{x}-\rangle\,\right]$$

then $|\hat{z}+;\hat{z}-\rangle$

$$= \frac{1}{2}\left[\,|\hat{x}+;\hat{x}+\rangle - |\hat{x}+;\hat{x}-\rangle - |\hat{x}-;\hat{x}+\rangle + |\hat{x}-;\hat{x}-\rangle\,\right]$$

etc...

$\Rightarrow |\text{"spin 0"}\rangle = \frac{1}{\sqrt{2}} \left[|\hat{x}+; \hat{x}-\rangle - |\hat{x}-; \hat{x}+\rangle \right]$

OR mixed i.e. #2 only as

$$|\hat{x}\pm\rangle = \frac{1}{\sqrt{2}} \left[|\hat{z}+\rangle \pm |\hat{z}-\rangle \right] \ldots$$

$\Rightarrow$ Quantum Mechanics Predicts that

(1) A measures S_z, B measures S_x

$\Rightarrow$ Completely random for each

(2) A measures S_x, B measures S_x

$\Rightarrow$ 100% Correlation!

$+ \rightarrow -$

$- \rightarrow +$

(3) A does nothing

$\Rightarrow$ B gets random results (50% $\pm$)

Question: Can a local Hidden Variable theory get the same answer?

YES

"Four Types of Particle Pairs"

$$\boxed{\begin{array}{l}(\hat{z}+,\ \hat{x}+)\\(\hat{z}+,\ \hat{x}-)\\(\hat{z}-,\ \hat{x}+)\\(\hat{z}-,\ \hat{x}-)\end{array}}$$

"White, Blue"
"White, Red"
"Black, Blue"
"Black, Red"

Large number N of pairs as follows:

Class	Number	Particle 1	Particle 2
1	$\rightarrow N/4$	$\hat{z}+, \hat{x}+$	$\hat{z}-, \hat{x}-$ "
2	$\rightarrow N/4$	$\hat{z}+, \hat{x}-$	$\hat{z}-, \hat{x}+$
3	$N/4$	$\hat{z}-, \hat{x}+$	$\hat{z}+, \hat{x}-$ "
4	$N/4$	$\hat{z}-, \hat{x}-$	$\hat{z}+, \hat{x}+$

Now look at the predictions!

(1) A measures $\hat{z}+$, Prob of B $\hat{x}+$ is

$$\frac{N_2}{N_1+N_2} = \frac{N/4}{N/2} = 50\% \checkmark$$

(2) A measures $\hat{x}+$, Prob B measuring $\hat{x}-$

$$= 100\%$$

(3) B alone, prob to measure $\hat{x}+$ is

$$\frac{N_2+N_4}{N} = \frac{N/2}{N} = 50\%$$

Great! Agrees with QM!

Enter J S Bell (1964)

Number	Particle 1	Particle 2	
N_1	$\hat{a}+, \hat{b}+, \hat{c}+$	$\hat{a}-, \hat{b}-, \hat{c}-$	
N_2	$\hat{a}+, \hat{b}+, \hat{c}-$	$\hat{a}-, \hat{b}-, \hat{c}+$	✓✓
N_3	$\hat{a}+, \hat{b}-, \hat{c}+$	$\hat{a}-, \hat{b}+, \hat{c}-$	✓✓✓
N_4	$\hat{a}+, \hat{b}-, \hat{c}-$	$\hat{a}-, \hat{b}+, \hat{c}+$	✓✓✓
N_5	$\hat{a}-, \hat{b}+, \hat{c}+$	$\hat{a}+, \hat{b}-, \hat{c}-$	
N_6	$\hat{a}-, \hat{b}+, \hat{c}-$	$\hat{a}+, \hat{b}-, \hat{c}+$	
N_7	$\hat{a}-, \hat{b}-, \hat{c}+$	$\hat{a}+, \hat{b}+, \hat{c}-$	✓✓
N_8	$\hat{a}-, \hat{b}-, \hat{c}-$	$\hat{a}+, \hat{b}+, \hat{c}+$	

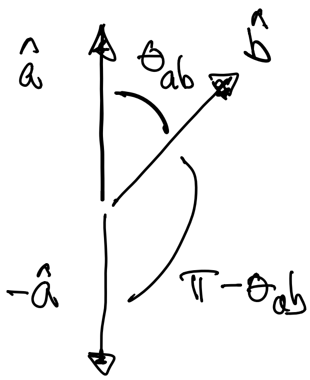
$$P(\hat{a}+, \hat{b}+) = \frac{N_3 + N_4}{N} \qquad N \equiv \sum_{i=1}^{8} N_i$$

$$\left.\begin{aligned} P(\hat{a}+, \hat{c}+) &= \frac{N_2 + N_4}{N} \\ P(\hat{c}+, \hat{b}+) &= \frac{N_3 + N_7}{N} \end{aligned}\right\} \quad \begin{aligned} &N_3 + N_4 \\ &\leq N_2 + N_3 + N_4 + N_7 \end{aligned}$$

$$\Longrightarrow P(\hat{a}+, \hat{b}+) \leq P(\hat{a}+, \hat{c}+) + P(\hat{c}+, \hat{b}+)$$

"Bell's Inequality"

What about Quantum Mechanics?



$P(\hat{a}+, \hat{a}-) = 1$

Rotate $\hat{a}-$ into $\hat{b}\,(+)$

Find component of $+\hat{a}$ in $\hat{b}$ direction as

$$\cos\left[\frac{\pi-\theta_{ab}}{2}\right] = \sin\frac{\theta_{ab}}{2}$$

See Prob. 1.11 (HW #2) or MQM3e (3.70)

$\leadsto P(\hat{a}+, \hat{b}+) = \cancel{\frac{1}{2}}\sin^2\frac{\theta_{ab}}{2}$

Sanity Check: $\theta_{ab} = 0 \Rightarrow P = 0$ ✓

$\theta_{ab} = \pi \Rightarrow P = \frac{1}{\cancel{2}}$ ✓

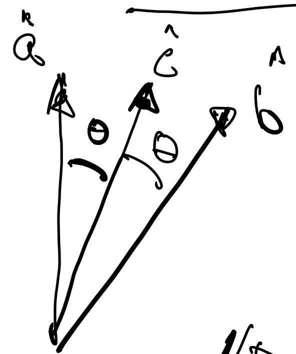
$\theta_{ab} = \pi/2 \Rightarrow P = \frac{1}{\cancel{4}\,2}$ ✓

Now check Bell's Inequality!

$P(\hat{a}+, \hat{c}+) = \frac{1}{2}\sin^2\frac{\theta_{ac}}{2}$

$P(\hat{c}+; \hat{b}+) = \frac{1}{2}\sin^2\frac{\theta_{cb}}{2}$

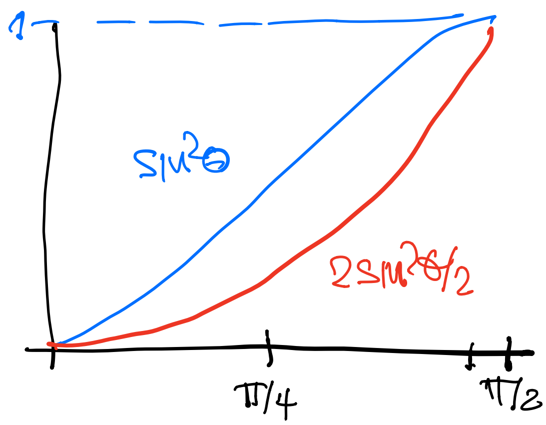
$$\Leftrightarrow \quad \sin^2 \frac{\theta_{ab}}{2} \overset{?}{\leq} \sin^2 \frac{\theta_{ac}}{2} + \sin^2 \frac{\theta_{cb}}{2}$$



special case!

$$\theta_{ab} = 2\theta \qquad \theta_{ac} = \theta = \theta_{cb}$$

$$\Leftrightarrow \quad \sin^2\theta \overset{?}{\leq} 2 \sin^2 \frac{\theta}{2}$$



VIOLATES BELL'S INEQUALITY FOR $0 \leq \theta \leq \pi/2$

$$\sin^2\theta = 4 \sin^2\frac{\theta}{2} \cos^2\frac{\theta}{2} \overset{?}{\leq} 2 \sin^2\frac{\theta}{2}$$

$$\Rightarrow \underline{\cos^2\frac{\theta}{2} \leq 1/2} \quad \text{only} \quad (\theta \geq \pi/2)$$

Can you do the experiment??

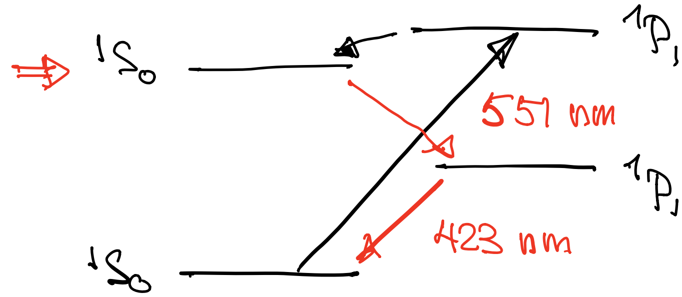
- $\pi^0 \to e^+e^-$ VERY RARE!
- $\eta \to \mu^+\mu^-$ HARD TO MEASURE P!

- Mercury Dimer: E. Fry (TAMU)

- Clauser (1969): Do it with photons!

↳ Freedman Clauser Experiment POSTED

Calcium Atoms:



↳ Bell's Inequality
is violated!

Agrees with quantum mechanics!