Detection of Cosmic Muons by their Deceleration and Decay

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Abstract

This study investigates the properties of muons, a fundamental particle often used in various scientific applications. By monitoring the photon release of the deceleration and decay of cosmic muons, the lifetime of muons was determined to be (2.203 ± 0.090) μs, in close agreement with the accepted literature value of 2.197 μs. Additionally, the Fermi Coupling Constant, G_F, which indicates the strength of the weak force that causes muon decay, was calculated to be $(1.162 \pm 0.024)10^{-5}$ GeV^{-2}, aligning closely with the literature-accepted value of $1.16638 * 10^{-5}$ GeV^{-2}. These measurements reveal the need for incorporation of the lifetime for any muon-based techniques such as muon probing of materials.

Introduction

The muon is one of the fundamental particles in the universe and can be thought of as an unstable heavier electron. The muon was first discovered in 1937 by C.W. Anderson and S.H. Neddermeyer. All properties of the muon are still not fully understood but despite this, the muon has played an integral role in the modern detection and imaging of the sub-surface dynamics of many systems. One of the first applications of muons was counting the number of incident muons into a volume and the number of muons leaving that volume to determine the total thickness of the material [1]. This type of technique became known as muon radiography. Another application combines this idea with scattering theory to create muon tomography. This involves using the scattering angle of muons to measure certain properties of the material being studied [1]. An example of this technique was modeling the time-dependent variation of the density of volcanoes using muon's energy spectra at different penetrating angles, which provided insight into volcanic activity [2]. As these involve techniques use collisions involving muons, the muon lifetime should be accounted for when taking spectra as the muon decay gives off energy. Muon detection is often done by detecting energy changes as these muons come from space and often have high kinetic energies so as they collide and decay, they give off energy as radiation. Measure the time between the collision pulse and decay pulse provides an estimate of the lifetime of the muon as their rapid deceleration removes any relativistic effects. Using this technique, the lifetime of muons was found to be (2.203 ± 0.090) μs.

Background and Theory

High-energy particles from space often collide with air molecules producing other types of particles. In particular, charged pions will decay either by interactions with air molecules or via the weak force to produce muons. After a muon is produced, the muon can strike another molecule and release its kinetic energy as a photon. After striking, the muon will then decay into an electron and neutrino via the weak force and release another photon. If at some initial time, suppose there are N_0 muons present. The muons decay exponentially and therefore the number of muons present at some later time t is as follows,

$$N(t) = N_0 e^{-t/\tau} \tag{1}$$

where τ is known as the lifetime of a muon. To find the decay probability after a certain time, differentiating Eq. 1 then dividing by the initial amount will give the probability of decay within a certain time interval dt.

$$D(t)dt = -dN/N_0 = \tau^{-1} e^{-t/\tau} dt \tag{2}$$

This implies that the characteristic lifetime of the muon, τ can be determined by modeling the time distribution of decays. The average muon has around 160 MeV of energy before collision. As they have such high energy, they travel at near-light speeds, and therefore due to the relativistic effect, barely any time has elapsed since their creation in their reference frame.

As muons decay from the weak force, the Fermi Coupling Constant, G_F, can aid in quantifying the strength of the weak force. The Fermi Coupling Constant can be calculated by the following equation,

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m^5 c^4} \tag{3}$$

Solving for G_F results in,

$$G_F = \frac{8\sqrt{3}\hbar^{7/2}\pi^{3/2}}{c^2 m^{5/2}\sqrt{\tau}} \tag{4}$$

Experimental Method

Figure 1 shows the apparatus for the experiment.

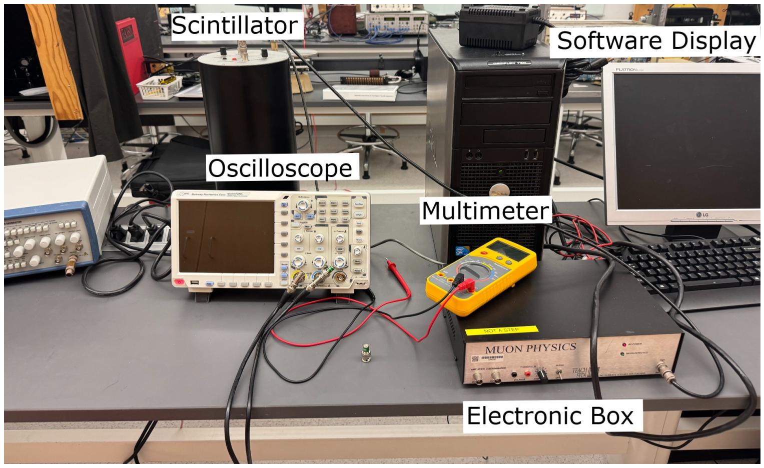

Figure 1: Apparatus for Muon Detection. The scintillator detects muon decelerations and decays to confirm the presence of a muon. The signals are sent to the electronic box where they are amplified and discriminated before arriving at the computer. The oscilloscope was used to measure the amplified and discriminator output while the multimeter was used to measure the threshold and the high voltage.

The electronic box took the input voltage from the detector and also output the amplification and the discriminator voltages. The scintillator worked as the detector for muons. Anytime a muon decelerated and decayed within the scintillator, it released a pulse to the electronic box that could sent to the software. The oscilloscope was connected to the output of the amplification and the discriminator to monitor and confirm that they were functional. The multi-meter was used to measure the discriminator threshold and the high voltage of the scintillator. The computer is connected to the electronic box to display the decay distribution graph and decay times of the muons.

Fig. 2 shows the circuit diagram for the apparatus.

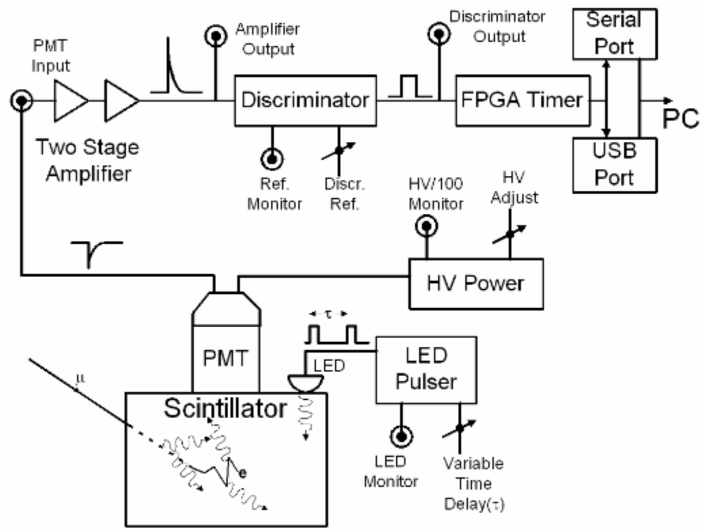

Figure 2: The Circuit Diagram of the Muon Apparatus. Photons from the scintillators are converted into electrical signals in the PMT which then gets amplified before going through the discriminator. If the signal passes the discriminator, the FPGA timer starts. If the timer is triggered again before 20 μs the event is counted as a muon.

This apparatus takes advantage of the two-step photon release of an incident muon. A muon will release a photon after colliding with the edge of the scintillator and will release another photon after it has slowed down and decayed. The time between these two events will be taken as a decay time for a muon and therefore a probability distribution can be plotted by the number of decays in a certain time interval. The amplifier ensures that all signals are detectable, while the discriminator ensures that the signals received are only signals caused by muons.

Results and Analysis

To ensure the equipment was working properly and ensure the muons could be measured accurately, a few exercises were done before the measure of muons. To begin, an alternating voltage was applied to the input of the electronic box, and the amplified output was measured. An input voltage was applied at a certain frequency and an amplified output was observed.

Table. 1 shows the amplified voltages at two different frequencies of the input voltage.

Frequency (KHz)	Input Voltage(mV)	Output Voltage (mV)
100.49	100	904
456.3	100	754

Table 1: Applified Output voltages at different Frequencies of the Input Voltage. An input peak-to-peak voltage of 100 mV was amplified at two different input frequencies. The 100.49 kHz was chosen as a lower frequency value while 456.3 kHz was chosen as it has a period of around 2 μs, the expected lifetime of the muon.

There was a decrease in the output voltage at a higher frequency. The amplifier increased the voltage by a factor of 9.04 and 7.54 for 100.49 kHz and 456.5 kHz respectively. As,

$$7.54/9.04 = 0.83$$

this implies that the signal only decreased by a factor of 0.83 as the frequency increased to what is expected from the muon, which ensures that muons can be reliably measured. An estimate for the minimum lifetime that could be observed is chosen to be a period of a frequency where the output voltage is a factor of 0.3 as compared to the output voltage at 100 kHz.

$$(904mV)(0.3) = 271mV$$

The frequency was then increased until an input of $100mV$ was amplified into an output peak-to-peak voltage of $271mV$. This occurred at a frequency of 2444.8 KHz or a period of 0.409 μs.

The maximum voltage amplification observed was 6.54 V at 100.49 kHz. Saturating the amplified output might cause two pulses to be counted as one pulse so the pulses cannot be too large to saturate the amp output but they cannot be too small to not pass through the discriminator. The discriminator gives a constant voltage but once a given input enters the discriminator, the voltage will fall to zero for as long as the voltage is above the threshold.

To observe the properties of the FPGA timer, the time between pulses was adjusted and compared to the times measured by the software. Table.2 contains the times measured by both the oscilloscope and the software.

Oscilloscope time (μs)	Software time (μs)
1.000	1.030
3.000	3.030
5.000	5.010
7.000	7.030
9.000	9.020
11.000	11.010
13.000	12.930
15.000	14.990
19.000	19.020
20.000	19.980

Table 2: Oscilloscope time as compared to Software time. The period of the pulses was set on the oscilloscope and the corresponding time on the software was reported. After 20 μs, the software did not record the time difference.

There were only slight differences in the oscilloscope time and software time of about 0.030 μs maximum. This shows strong agreement and precision for the times between pulses.

The software does not record any times greater than 20 μs, to find the minimum frequency, the pulses were set to have a minimum time delay of 0.640 μs. These two pulses were still registered as separate events and therefore, it is known that any time greater than this minimum time delay can be registered. The bin width of the time spent must be between 0 and 640 ns.

To find the best setting for the discriminator, it was set to 102.3 mV and the high voltage was set to 1,100 V. The detector ran for 45 hours with 818,000 muons detected but only 10,000 decays. There was much more decays in the first 0.5 μs than expected. This was most likely due to double pulsing from the PMT and so the discriminator was 241.4 mV with a high voltage of 1,019 V to try to prevent the secondary pulses from registering. This setup was run for 45 hours with 8,000 decays. Fig. 3 contains the graph of the binned number of events over time.

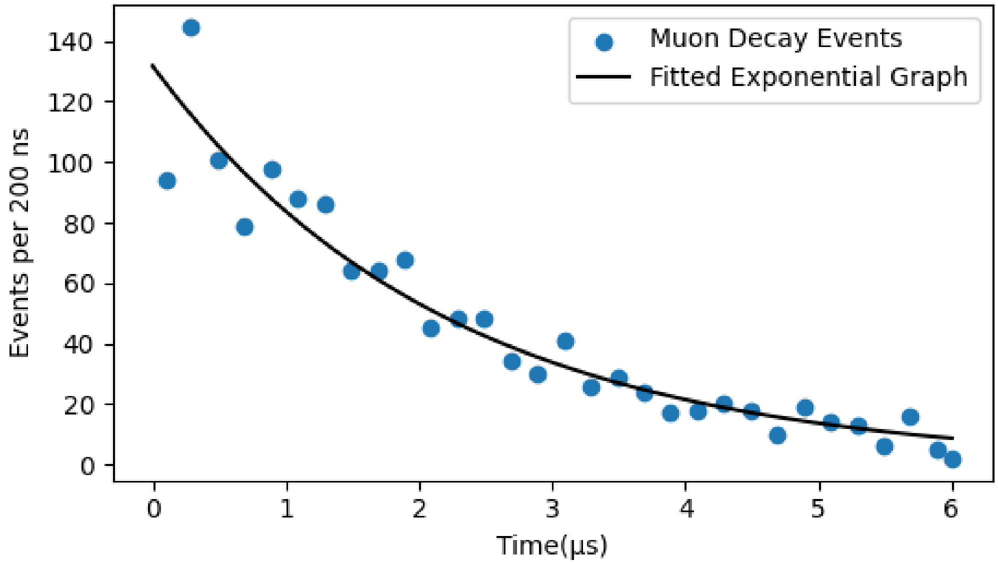

Figure 3: Binned Muon Decay Data as a function of Time. The muon decay events were binned in 200 ns intervals. The binned data was fit to an exponential decay curve to determine the muon lifetime.

The data was fitted to an exponential decay which resulted in the following equation,

$$N(t) = (131.7594)e^{t/2.203\mu s} \tag{5}$$

where $N(t)$ is the number of decays after a time t. The calculated lifetime of the muon is $\tau = 2.203$ μs ± 0.090 μs. The accepted-literature value of 2.19698 μs results in an error $\approx$ 0.27% [?]. To calculate the fermi coupling constant, Eq. 4 was used. The value is usually reported in units of GeV^{-2} so the value is divided by $(\hbar c)^3$, therefore the calculated value for the Fermi Coupling Constant is, $G_F/(\hbar c)^3 = (1.162 \pm 0.024) * 10^{-5}$ GeV^{-2}. The accepted value is $G_F/(\hbar c)^3 = 1.16638 * 10^{-5}$ GeV^{-2} [?]. There is an error $\approx$ 0.26% difference between the calculated value and the literature-accepted value.

Conclusions

The muon lifetime was determined to be $(2.203 \pm 0.090)\mu s$ with an error of about 0.27% as compared to the literature value. This was determined through the deceleration of the muon to eliminate the relativistic effects so that its lifetime could be measured in our frame of reference to an accurate degree. Knowing the lifetime of the muon ensures that the application of these cosmic muons can account for the weak force decay. As the muons will come into contact with these materials they will rapidly lose their energy and emit photons, which can be observed that can help characterize these properties such as composition or density. Future studies could delve into the energy spectra of the light given off by the muons depending on the density or makeup of materials hit by muons.

Appendix

Calculations

Muon lifetime (τ)

$$\frac{2.203 - 2.19698}{2.19698} = 0.27\%.$$

Fermi Coupling Constant (G_F)

$$G_F = \frac{8\sqrt{3}\hbar^{7/2}\pi^{3/2}}{c^2 m^{5/2}\sqrt{\tau}}$$

$$\frac{G_F}{\hbar^3 c^3} = \frac{8\sqrt{3}\hbar^{1/2}\pi^{3/2}}{c^5 m^{5/2}\sqrt{\tau}}$$

$$\frac{G_F}{\hbar^3 c^3} = \frac{8\sqrt{3}(1.05457 * 10^{-34}Js)^{1/2}\pi^{3/2}}{(2.998 * 10^8 m/s)^5 (1.884 * 10^{-28}kg)^{5/2}\sqrt{2.203 * (10{-}6)s}} = 4.528 * 10^{14} J^{-2} = 1.162 * 10^{-5} GeV^{-2}$$

Error in Fermi Coupling Constant (δG_F)

$$\frac{dG_F}{d\tau} = \frac{-4\sqrt{3}\hbar^{7/2}\pi^{3/2}}{c^2 m^{5/2}(\tau)^{3/2}}$$

$$\delta G_F/\hbar^3 c^3 = \frac{dG_F/\hbar^3 c^3}{d\tau}\delta\tau = \frac{4\sqrt{3}(1.05457 * 10^{-34}Js)^{1/2}\pi^{3/2}}{(2.998 * 10^8 m/s)^5 (1.884 * 10^{-28}kg)^{5/2}(2.203 * (10{-}6)s)^{3/2}}(0.090 * 10^{-6}s)$$

$$= 9.2 * 10^{12} J^{-2} = 2.4 * 10^{-7} GeV^{-2}$$

$$\frac{1.16638 - 1.162}{1.6638} = 0.26\%$$

References

[1] G. Bonomi, P. Checchia, M. D'Errico, D. Pagano, and G. Saracino, "Applications of cosmic-ray muons," *Progress in Particle and Nuclear Physics*, vol. 112, p. 103768, 2020.

[2] H. Tanaka, K. Nagamine, N. Kawamura, and et al., "Development of the cosmic-ray muon detection system for probing internal-structure of a volcano," *Hyperfine Interactions*, vol. 138, pp. 521–526, 2001.