Single Photon Interference and Diffraction

Jacob Shin*

Temple University

Department of Physics

The 2-slit experiment was performed using both many-photons at a time, using a laser and photodiode detector, and one photon at a time using a light bulb and pulse counter. The intensity was measured as a function of the position on the detector screen using a detector slit. The voltage of the photodiode was used to find the intensity for the many-photon experiment while the photon counts in a 10 s interval was used for the intensity in the single photon experiment. We confirmed that the 2-slit interference pattern with one photon at a time was similar to the many photon pattern and matched the patterns given by the Fraunhofer model, showing the quantum nature of light. We used the interference pattern to find the laser wavelength, $\lambda = (632 \pm 7)$ nm, and light bulb wavelength, $\lambda = (530 \pm 70)$ nm. Finally, we confirmed that the photon counts in a given 0.1 interval follow a Poisson distribution with $\mu = 183$ counts while the time in between photons follows a waiting time distribution with probability rate, $\lambda = 0.00188\ \mu\text{s}^{-1}$.

I. INTRODUCTION

Performing the classic 2-slit and 1-slit experiments using one photon at a time was important since it confirmed that interference and quantum phenomena still occur with just one photon. Then 2-slit and 1-slit experiments used double slits and single slits, respectively, with apertures much smaller than the distance between the detector and slits. When light passed through the slits, instead of simply casting a shadow like the particle model would predict, there was a pattern with minima and maxima. This allowed physicists to observe the "wave" nature of light. With many photons, interference and diffraction can be explained semi-classically using waves and the Fraunhofer model. However, when sending photons one at a time still leads to the same interference/diffraction patterns, this shows that one photon is "interfering" with "itself," showing the true quantum mechanical nature of light.

We used a laser and photodiode to get the intensity of light reaching the detector to get the 2-slit interference pattern with many photons coming at a time. This confirmed the wave nature of light when there were many photons coming through the slits at the same time.

We then used a light bulb and filter that only lets green light through to get photons reaching the slit one at a time. The light bulb was used since it mainly emits red light at low power settings. The use of the filter reduces the number of particles to the point that 0.3% there is one photon in the apparatus and 99.7% of the time there

* jacob.shin@temple.edu

are no photons [1]. We discovered a similar 2-slit interference pattern compared to the many photon method. This interference pattern could not be simply explained as the sum of the two 1-slit diffraction patterns, showing that the classical method of superposition was not the cause of the interference pattern.

II. THEORY

A. Interference and Diffraction

We performed both interference (2-slit) and diffraction (1-slit) experiments.

The intensity, I, as a function of position on the detector screen, y, can be derived using the Fraunhofer model. This assumes that the distance between the aperture and detector is orders of magnitudes greater than the size of the aperture, allowing us to treat the incident light as plane waves.

In the 2-slit interference geometry, the intensity at the screen is

$$I_2(\theta) = I_0 \left(\frac{\sin\alpha}{\alpha} \right)^2 (\cos\beta)^2 \tag{1}$$

and for 1-slit diffraction

$$I_1(\theta) = \frac{I_0}{4} \left(\frac{\sin\alpha}{\alpha} \right)^2 \tag{2}$$

where

$$\alpha = \frac{\pi a}{\lambda} \sin\theta, \qquad \beta = \frac{\pi d}{\lambda} \sin\theta \tag{3}$$

and where I_0 is the intensity at the central maximum, θ is the angle between the central maximum and the location on the detector screen, a is the aperture size, d is the slit spacing, and λ is the wavelength of incident light. These equations can be used to describe the patterns that will be measured via the detector.

B. Poisson Distribution

The Poisson distribution is used for processes where the probability of an event occurring in the future is independent of what happened in the past. This property is called memoryless. This distribution is used to describe the production of photons for the one photon at a time experiment. The Poisson distribution is used when there is a large number of events, but the probability of an event occurring is low. In this experiment, there is a large number of photons produced by the light bulb, but only a few green photons get past the filter, meaning a low event probability.

A Poisson distribution has the following properties:

$$\mu = \lambda = \sigma^2 \tag{4}$$

where μ is the mean, σ^2 is the variance, and λ is the parameter that characterizes a Poisson distribution. λ is a measure of the probability per unit time of an event occurring [2].

The probability distribution function (pdf) has the form:

$$P(k;\mu) = e^{-\mu}\frac{\mu^k}{k!} \quad \text{for } k = 0, 1, 2, \ldots. \tag{5}$$

The pdf can be used to show the results of Eq. (4)

$$\langle k \rangle = \sum_{k=0}^{\infty} kP(k;\mu) = \mu \quad ; \tag{6}$$

$$\sigma^2 = \left\langle k^2 \right\rangle - \langle k \rangle^2 = \sum_{k=0}^{\infty} k^2 P(k;\mu) - (\mu)^2 = \mu(\mu+1) - (\mu)^2 = \mu. \tag{7}$$

C. Waiting Time/Exponential Distribution

The waiting time distribution is used to describe the time in between events that have a constant rate (probability per unit time) of occurring. This is used to characterize the amount of time in between single photons arriving at the detector in Section IV B.

The probability distribution function (pdf) of the exponential distribution is

$$\rho(T;\lambda) = \lambda e^{-\lambda T}. \tag{8}$$

where λ is still the probability per unit time of the event occuring.

The mean μ is

$$\mu = \langle T \rangle = \int_0^{\infty} T\rho(T;\lambda)dT = \frac{1}{\lambda}. \tag{9}$$

This shows that the mean is just the reciprocal of the rate of events.

The variance, σ^2, is

$$\sigma^2 = \left\langle T^2 \right\rangle - \langle T \rangle^2 = \int_0^{\infty} T^2\rho(T;\lambda)dT - \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}. \tag{10}$$

Similar to the Poisson distribution, the exponential is memoryless and the mean and variance don't depend on when one starts waiting/counting.

III. EXPERIMENTAL METHOD

A. Equipment

- Sources
  - Red Laser Diode
  - Light Bulb with a filter that only passes green light
- Source Slit (Singe Slit)
- Double Slit (0.353 mm Slit Spacing)
- Slit Blocker (Wide Slit) with Micrometer
- Detector Slit (Single Slit) with Micrometer
- Detectors
  - Photodiode
  - Photomultiplier with Pulse Counter and Interval Timer (PCIT)

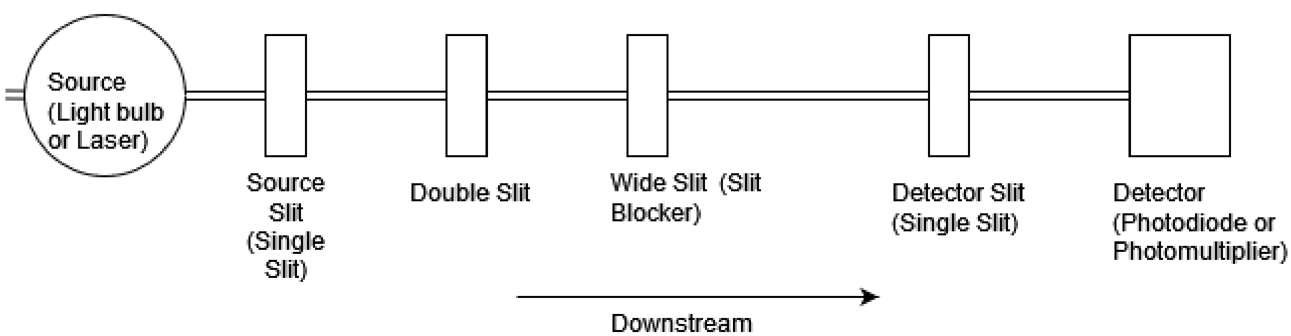


FIG. 1: Setup of apparatus.

The setup for the experiment is shown in Figure 1. Light is produced at the "source," which goes to the right (downstream) and eventually goes through a double slit. The slit blocker can be configured to let light through one or both slits for single slit diffraction and double slit interference experiments, respectively. The diffraction or interference pattern makes it to the detector. The single source slit and single detector slit are used to remove extraneous light.

The apparatus can be configured to either use a laser light source or light bulb source with a filter that only lets green light through. The laser source will allowed for the experiments with multiple photons present at a time while the light bulb can allowed for single photon measurements. Depending on the position of the slit blocker, either the 1-slit or 2-slit experiments were performed.

The source can be switched out between the laser and light bulb. Only one source was used at a time. The laser was used for experiments using multiple photon at a time, while the light bulb was used to get single photons at a time. The source slit has a single slit (on the order of a tenth of a mm) that allows the light coming out of the source to be a narrower beam by removing extraneous light. The double slit has two slits with slit spacing, $d = 0.353$ mm, and slit width, $a \approx 0.1$ mm. When both slits were unblocked and open, the 2-slit experiment was performed. The 1-slit experiment is performed by using the slit-blocker to block one of the slits on the double slit. The slit blocker has one slit several orders of magnitude wider than the double or single slits and can be adjusted with an attached micrometer. The detector slit is a single slit which can be adjusted with a micrometer. It was used to select which part of the interference or diffraction pattern reaches the detector. Finally, the detector used either a photodiode, which gives the intensity of the light as a voltage readout, or a photomultiplier, which can be used to measure single photon events. The photomultiplier needed to be connected to the pulse counter to detect single photon events. The detector also has a shutter that can be closed to prevent light from reaching the photomultiplier.

B. Laser Photodiode Two Slit Measurement

The laser generates a large number of monochromatic photons which can be detected via a photodiode. For the 2-slit experiment, the slit blocker was set so that none of the slits on the double slits were blocked. The detector slit position was varied to select different parts of the interference pattern. At each detector slit position, the voltage of the photodiode was measured to get the intensity for the interference pattern.

C. Slit Experiments with One Photon at a Time (Pulse Counter)

The bulb with a filter was used as a source since it allowed for only one photon to enter the apparatus at a time. The pulse counter was used with the photomultiplier to measure these single photon events. The bulb power was set to 5, the discriminator to 2.95, and the high voltage bias of the photomultiplier to 900 V. These setting were reached primarily through trial and error to get the highest signal to noise ratio (counts while shutter open vs. shutter closed). We found a signal to noise ratio of $\mathrm{S/N} = \frac{9652}{492} = 19.6$. We measured the counts over a 10 s interval for each position of the detector slit. The 2-slit experiment was performed in the same way as described above. The 1-slit experiment was done in a similar way except that the slit blocker was moved to a position so that one slit on the double slit was blocked. The 1-slit experiment was performed two times: one where the "far" slit on the double slit was open and the other where the "near" slit was open. Near and far correspond to the face of the apparatus parallel to the optical axis if the source and detector go from left to right while facing the experimentalist.

IV. RESULTS AND ANALYSIS

A. Laser Photodiode Two Slit Experiment

TABLE I: Fit parameters found for 2-slit experiment using the laser and photodiode detector. The model for 2 fit interference follows the form of $a \cdot \text{sinc}^2\left(\frac{y-d}{b}\right)\cos^2\left(\frac{x-d}{c}\right)$ where a, b, c, and d are fit parameters determined by Matlab's fit function. The uncertainty in fit parameter indicates the 95% confidence interval.

a	b	c	d
0.90 ± 0.03	3.76 ± 0.17	0.294 ± 0.002	3.971 ± 0.006

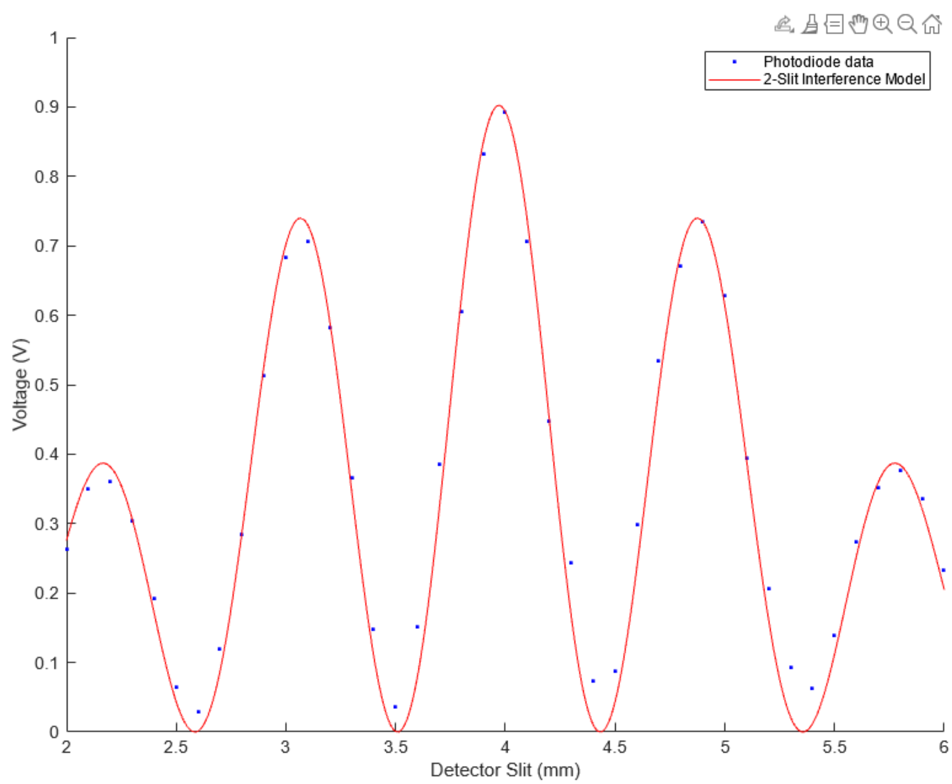


FIG. 2: Photodiode voltage vs. detector slit position and model fit curve. The fit curve uses the 2-slit interference equation from Eq.(1). The fit parameters are in Table I.

TABLE II: Location of intensity maxima for the 2-slit interference experiment. The location corresponds to the detector slit micrometer measurements in mm. m_c is the location of the central maximum while m_{li} is the location of the i-th closest maximum to the left of the central maximum. m_{1r} is the first closest maximum to the central maximum on the right while m_{2r} is the second closest to the central maximum on the right side.

m_{2l}	m_{1l}	m_c	m_{1r}	m_{2r}
2.20	3.08	4.01	4.91	5.81

Figure 2 gives the relationship between intensity of the 2-slit interference pattern and detector slit position. A fit curve based on the interference Fraunhoffer model using Matlab to fit the parameters is found in Table I. Table II gives the position of the central maximums and surrounding maximas. The average spacing between maximums is

$$\overline{\Delta y} = \frac{0.88 + 0.90 + 0.93 + 0.91}{4} = 0.90 \pm 0.01 \tag{11}$$

where the uncertainty was calculated as the standard deviation of the mean (SDOM) [3].

The wavelength of the laser source can be calculated using this spacing. We have the following relationship between θ, the angle between the central maximum and the maximum of interest on the interference pattern; y, the vertical distance from the central maximum to maximum of interest, and L, the distance from the double slit to the detector:

$$\tan\theta = \frac{y}{L} \tag{12}$$

We also have the following relationship between λ, the wavelength of the source light; d, the spacing between the two slits; n the n-th closest maximum to the central maximum, and θ:

$$\frac{\lambda}{d} = \frac{\sin\theta}{n} \tag{13}$$

with $d = 0.353$ mm, $L = 503$ mm, $n = 1$, and $y = 0.90 \pm 0.01$ mm, we get $\lambda = (632 \pm 7)$ nm. This was done through standard error propagation rules [3]:

$$\begin{aligned}
\Delta\lambda &= \sqrt{\left(\frac{\partial\lambda}{\partial y}\Delta y\right)^2} \\
&= \sqrt{\left(\left(\cos\left(\arctan\left(\frac{y}{503}\right)\right)\frac{0.353}{503}\frac{1}{(y/503)^2+1}\right)\cdot\Delta y\right)^2} \\
&= 7 \times 10^{-9}
\end{aligned}$$

$$\lambda = (6.32 \pm 0.07) \times 10^{-7}$$

The expected value is 670 nm, but measured value of 632 nm is still red light. The apparatus may have shifted slightly during the experiment, leading to some additional error. The micrometer used to measure the positions is also prone to "slack" where there are slight differences in the measurements depending on which direction one turns the micrometer.

B. Slit Experiments with One Photon at a Time (Pulse Counter)

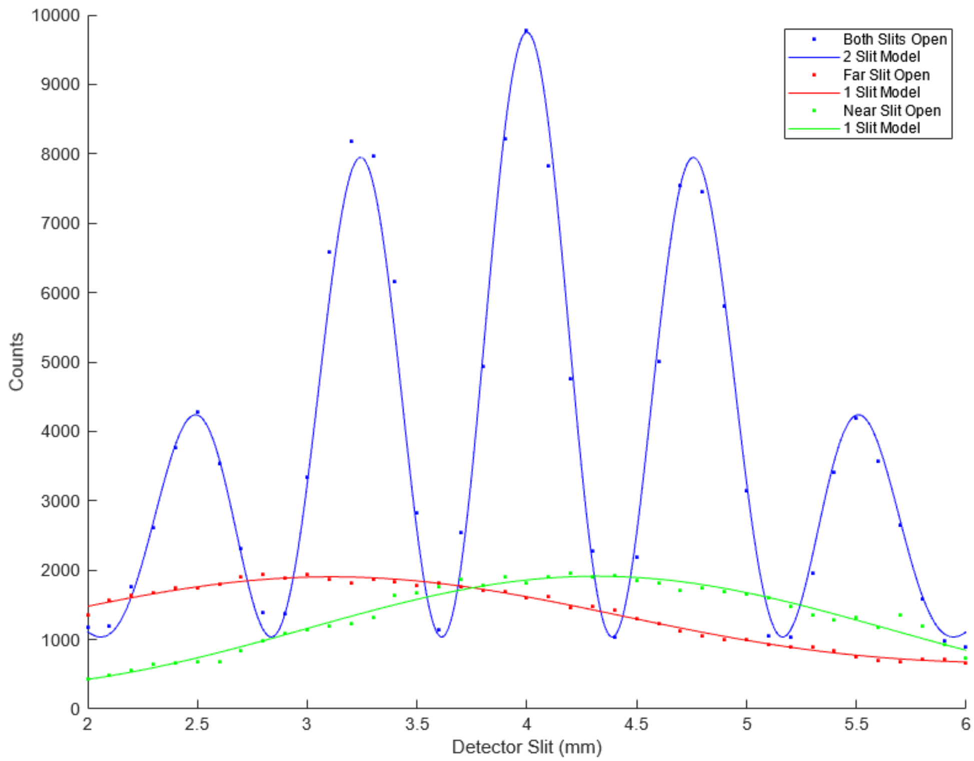


FIG. 3: Plot of raw data of counts vs. detector slit position and fits based on the diffraction and interference models for 1-slit and 2-slit experiments. When "both slits open" on the slit blocker, the 2-slit interference pattern is measured. For the "far slit open" and "near slit open," the 1-slit diffraction pattern is measured since only one slit on the slit blocker allows light through. The curve fit models are based on Eq. 1 and Eq. 2 for 2-slit inteference and 1-slit diffraction, respectively. Table III has the fit parameters found in the experiment.

TABLE III: Fit parameters found for when the slit blocker has the far slit open, near slit open, and 2 slits open. The model for 2 fit interference follows the form of $a \cdot \text{sinc}^2\left(\frac{y-d}{b}\right)\cos^2\left(\frac{x-d}{c}\right) + e$ where a, b, c, d, and e are fit parameters determined by Matlab's fit function. Similarly, the 1-slit diffraction model (far and near slit open) uses the equation of the form $a \cdot \text{sinc}^2\left(\frac{y-d}{b}\right)$, with a, b, d, and e being fit parameters. The uncertainty in fit parameter indicates the 95% confidence interval.

	a	b	c	d	e
Far Slit Open	1240 ± 50	3.2 ± 0.2		3.11 ± 0.04	670 ± 60
Near Slit Open	1700 ± 300	3.2 ± 0.5		4.32 ± 0.04	250 ± 280
Two Slits Open	8700 ± 300	2.9 ± 0.1	0.247 ± 0.002	4.000 ± 0.006	1040 ±160

Figure 3 shows the intensity as a function of detector slit position for the 2-slit and 1-slit experiments using one photon at a time in the apparatus. "Both Slits Open" is the 2-slit experiment while the "Far Slit Open" and "Near Slit Open" are both the 1-slit experiment. A fitting curve is used for all 3 based on Eq. (1) and (2). The fitting parameters from Matlab's fit function are recorded in Table III.

The wavelength of the light bulb source can be calculated in a similar way as in Section IV A. We found that the average spacing between maxima was $y = (0.75 \pm 0.10)$ mm, which leads to $\lambda = (530 \pm 70)$ nm. This is close to the expected value of 550 nm provided by the manufacturer. 530 nm also corresponds to green light.

We see that even when we add up the values of the far slit and near slit curves, the curve with both slits open still has maxima that exceed the sum. Thus we see interference still occurs even with just one singular photon for when both slits are open.

C. Pulse Counter and Interval Timer Statistics

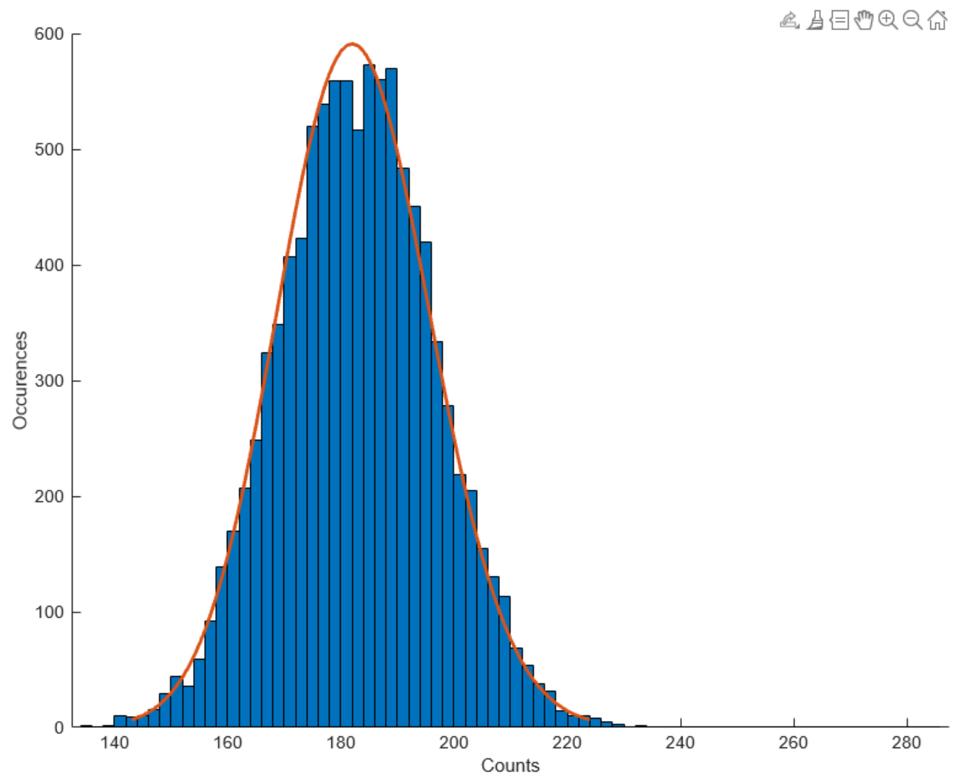


FIG. 4: Histogram of Counts in 0.1 s. The number of photons was measured during a 0.1 s interval. This 0.1 interval measurement was made 10,000 times. A Poisson distribution fit curve was plotted over the histogram. The mean of the counts was $\mu = 183$ and variance $\mathrm{Var}(X) = 189$.

In Figure 4, there is a histogram of multiple 0.1 s interval count measurements. The production of photons is random and matches a Poisson process due to the independent nature where the generation of a photon at one point in time doesn't affect the photon at a later time. Thus, a Poisson distribution curve is fitted over the histogram data.

According to Eq. 4, the mean and variance should be the same. For the counts data, $\mu = 183$ and $\sigma^2 = 189$, which are close as expected.

Figure 5 shows the distribution for waiting times, the time between each photon detection event when using the one photon at a time setup. We have $\mu = 531$ μs and $\sigma^2 = 2.87 \times 10^5$ μs.

We expect that

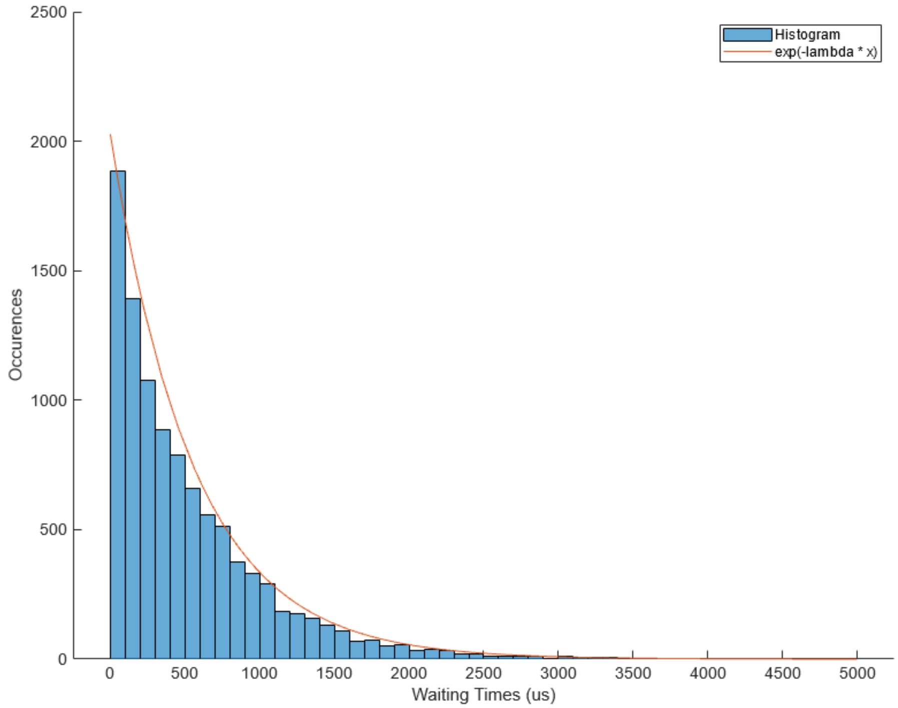
$$\mu = \frac{1}{\lambda} \tag{14}$$

and

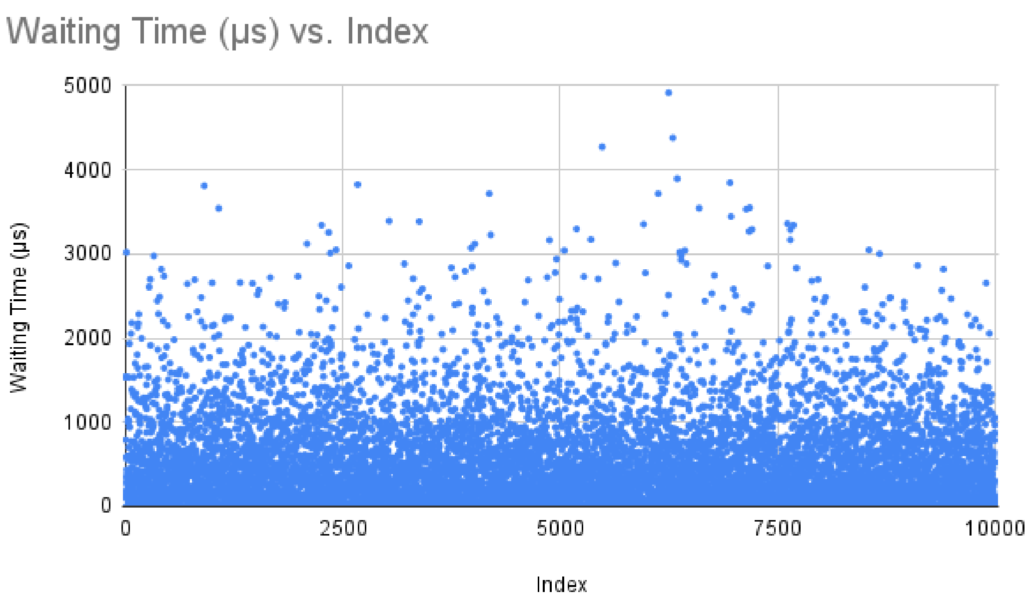
$$\sigma^2 = \frac{1}{\lambda^2} = \mu^2 \tag{15}$$

The square of the mean is $\mu^2 = 2.82 \times 10^5$ us, which is close to the variance of $\sigma^2 = 2.87 \times 10^5$ us.

The probability rate is then $\lambda = \frac{1}{\mu} = 0.00188$ μs^{-1}.



(a) Histogram of Waiting Times



(b) Scatter Plot of Waiting Times vs. Index

FIG. 5: (a) The histogram of waiting times follows an exponential decay, which corresponds to a waiting time distribution (See Section II C). (b) The scatter plot lists the points (j, C_j) where C_j is the amount of waiting time for the j-th photon detection event. This scatter plot fits the qualitative description described in the experiment manual since the points look like particles in a gravitationally bound atmosphere where there are less particles at higher altitudes [2]. The waiting distribution is characterized by the parameter, $\lambda = 0.00188\ \mu\text{s}^{-1}$. This was determined by taking the mean of the waiting times $\mu = 531\ \mu$s (See Section II C). The variance was $\text{Var}(X) = 2.87 \times 10^5$ μs.

V. CONCLUSIONS

We first used a laser and photodiode to perform the traditional 2-slit experiment with many photons at a time. This yielded the expected interference pattern based on Eq. (1). We also found the laser wavelength using the interference pattern $\lambda = (632 \pm 7)$ nm. We then used a light bulb to get an interference pattern even when making measurements one photon at a time. Interference occurs since the pattern cannot be explained simply as the sum of the two 1-slit diffraction patterns. The wavelength of the filtered light bulb was measured as well $\lambda = (530 \pm 70)$ nm. Finally, we performed measurements of the probability distributions related to the production of single photons. We found that the counts of photons in a 0.1 s interval followed a Poisson distribution with $\mu = 183$ μs and $\sigma^2 = 189$ μs. The time between the arrival of photons followed an exponential distribution instead with the probability rate of $\lambda = 0.00188$ μs^{-1}.

[1] *TeachSpin's Two-Slit Interference, One Photon at a Time A Conceptual Introduction to the Experiment*.

[2] *TWO-SLIT INTERFERENCE, ONE PHOTON AT A TIME*, TeachSpin, Inc., Buffalo, NY, rev 2.0 6/2013 ed. (2013).

[3] J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, 1996).