Experiments with Radioactivity

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Radioactivity is a fascinating phenomenon, rooted in quantum mechanics, which also lends itself to the study of purely statistical phenomena. You will investigate both of these aspects with several different measurements in this laboratory exercise.

Radioactive Decay

Some atomic nuclei are "unstable" and undergo radioactive decay. The most common mode of decay is β^- where a neutron in the nucleus changes into a proton and emits an electron and neutrino. The "daughter" nucleus is generally in an excited state which emits high energy photons, called "γ-rays", as it decays to the ground state. The quantized energy levels of nuclei are typically measured in MeV, so γ-ray photons are typically 10^{4-5} times more energetic than the optical photons emitted in atomic transitions.

The interaction responsible for β-decay is much weaker than the electromagnetic interaction which leads to atomic transitions. (Indeed, we call it the "Weak Interaction.") This means that the probability of any one nucleus decaying is much smaller than what we are used to seeing in atoms. In fact, it is so small that given a large number of radioactive nuclei, we can actually measure the "lifetime" by watching the atoms decay over, say, minutes or hours. Furthermore, it is not uncommon that the quantum numbers of excited nuclear states strongly hinder electromagnetic transitions, so some of the γ-ray emissions also have long lifetimes. (Such states are called "isomers.")

If we start with a sample containing a very large number N_0 of radioactive nuclei at time $t = 0$, the number $N(t)$ left at time t will decrease at a rate R proportional to N, assuming the probability of decay is constant and does not change with time. We write this as

$$R = \frac{dN}{dt} = -\lambda N \tag{1}$$

where λ is called the "decay constant." The solution to this differential equation is simple, namely $N = N_0 e^{-\lambda t}$, hence the term "exponential decay." The decay rate is then

$$R = -\lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} = R_0\, 2^{-t/t_{1/2}} \tag{2}$$

where $t_{1/2} = \ln 2/\lambda$ is called the "half life." That is $t_{1/2}$ is the time it takes for the one-half of the sample to decay. (Sometimes we refer to the "lifetime" $\tau = 1/\lambda$.)

The decay rate R, also called the "activity," of a sample is historically measured in micro Curies (μCi). One Curie is defined to be 3.7×10^{10} decays per second (Bq), so 1μCi=3.7×10^4 Bq. The SI unit of activity is the Becquerel (Bq).

In this lab, you will measure both β^- decays and the γ-ray emissions that accompany them. You'll learn some details about detecting radioactivity with a Geiger-Müller counter, demonstrate the fundamentally random nature of these emissions, make some measurements of the characteristics of β and γ radiations, and observe the ≈ 2.5 minute half life of an excited isomeric state in the ^{137}Ba nucleus.[1]

The Apparatus

The radiation detector is a Geiger-Müller (GM) counter, Frederiksen model 5135.70. A GM counter works by having the radiation ionize atoms in a gas volume, then accelerating the electrons (and ions) through a ~ 500 V potential difference. The radiation particles enter the gas volume through a thin window with an effective diameter of 9 mm. A GM counter is nearly 100% efficient for detecting β particles, but only a few percent efficient for γ-rays. (Why?) The signal is amplified because the electrons create an "avalanche" resulting in a current pulse on the anode, so each electronic pulse measures a single particle of radiation.

The pulses are recorded using the standard Pasco interface and software, allowing you to record the number of decays over some controllable time period, repeating over and over. This is easily turned into a rate as *detected* pulses per second.

You have a collection of radioactive sources, namely ^{90}Sr which emits β's, and ^{60}Co and ^{137}Cs γ-ray sources. In all cases the energy of the emitted particles are on the order of an MeV. **You should look up the decay schemes of these nuclei and include figures in your lab report.** Note that each source specifies the activity in μCi along with the half life and when the source was created. The activity is concentrated in a spot at the center of the plastic disk.

The sources are placed in little trays that are slid into a holder underneath the window of the GM counter. The holder has several slots, so you can place the source nearby or relatively far away from the detector.

Start by getting used to the apparatus. Pick one of the sources and put it on a tray near and then farther from the detector, and watch the rate change. This will also help you come up with standards for taking and recording data.

One measurement you should make is the "background" rate of the detector with no sources nearby. It will still give some counts, due to cosmic radiation and ambient radiation in the environment, both of which are very difficult to shield away. Remember that whenever you measure a rate with a source, this background rate needs to be subtracted.

[1] You should recall from your Modern Physics class that we label nuclei as ^AZ where A is the mass number and Z is the atomic number, labeled by its chemical element name.

Measurement: Estimating the Decay Rate

Using one of the ^{90}Sr sources, carefully measure the detected rate as a function of distance from the GM counter window. Verify that the rate falls with distance as you expect, based on the subtended solid angle. (Calculating the solid angle of a flat, circular detector is not hard; see Melissinos and Napolitano Section 9.1, for example.)

Now use your result to calculate the activity of the source, and verify the value on the label. Don't forget to account for how much of the source has decayed away since it was created.

Repeat this with one of the ^{60}Co sources. Don't forget about the different detection efficiencies for β and γ particles.

Don't expect that you'll be able to make a very precise measurement for comparison to the label. Can you think of reasons why there will be large systematic uncertainties?

Measurement: Random Statistics and the Poisson Distribution

Radioactive decay is the prototypical random process. According to quantum mechanics, it is impossible to determine for certain whether or not a particular nucleus will decay within a certain time span. It is only possible to calculate the probability that it will decay.

Radioactive decay is also the prototypical example of the Poisson probability distribution. The Poisson distribution arises when you have a process which is extremely improbable, for example the decay of any one particular nucleus, but you have a very large number of chances that it might happen, that is the very large number of nuclei is a macroscopic sample.

According to the Poisson distribution, the probability of observing n events (eg decays) when the average number of decays is μ is given by

$$\mathcal{P}_\mu(n) = \frac{\mu^n}{n!}e^{-\mu} \tag{3}$$

which obviously implies that $\sum_n \mathcal{P}_\mu(n) = 1$ as it must be. An important property of this distribution is that the standard deviation of a set of measurements is $\sigma = \sqrt{\mu}$.

Test this by using your data from estimating the decay rate, or generate new sets of data. Pick at least three different configurations where the mean number of counts is not much than one, somewhat more than one, and much more than one. In each case, use the average of the number of counts to approximate μ and compare $\sqrt{\mu}$ to the standard deviation of the data set. Make histograms of the number of counts, and compare to a plot of Equation 3.

For your data set with the largest mean, show that the distribution is also very well approximated by the Gaussian distribution, namely

$$\mathcal{G}_{\mu,\sigma}(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(n-\mu)^2/2\sigma^2} \qquad \text{where} \qquad \sigma = \sqrt{\mu} \tag{4}$$

Measurement: Passage of Radiation through Matter

The goal here is to get an understanding of the relative penetrating nature of β- and γ-radiation. You'll not be able to make a precise determination, but the difference between the two effects should be very clear.

Set up the apparatus with a tray somewhere in the middle of the holder. Use a tray in the top slot to hold various pieces of lead, plastic or other materials that you'll find with the setup. Watch the rate decrease as you increase material and try to identify how much you need to make the signal disappear.

Do this for one each of the ^{90}Sr, ^{60}Co, and ^{137}Cs sources. Try to find a reference that will help you verify your answer, although doing this to significant precision is very difficult. Can you see differences between the ^{60}Co and ^{137}Cs sources? It will be helpful for you to do a little outside reading on the characteristics of the radiations from these three sources.

Measurement: Exponential Decay

A kit is provided which allows you to use a weak acid solution to chemically separate the ^{137}Ba daughters from a sealed ^{137}Cs source. Use the syringe to draw in a few CC's of the soluition, and then squeeze a few drops through the source into one of the trays. Put the tray in the top slot and start counting. You will see the rate decay exponentially over a few minutes, on top of some background.

Repeat a few times to get the optimal signal-to-background ratio. (You might want to experiment with how fast you should squeeze.) Note that if you use an especially short time window for counting, you can always combine bins later when you are analyzing it.

Use the software of your choice to perform a nonlinear three-parameter fit to the function

$$R(t) = R_0\, e^{-\lambda t} + B$$

to determine the value of λ and compare to the known value of the half life of the ^{137}Ba isomer. You should do this with weights based on the uncertainties on the number of counts in any time bin equal to the square root of the number of counts. (This is Poisson statistics, after all.) Use the appropriate software to determine the uncertainty on the half life as well its value.