Voltage Dividers and Passive Filters

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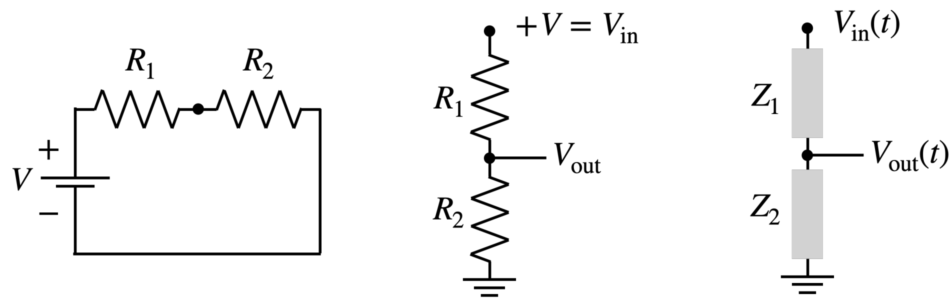
In this experiment you will learn how to use an oscilloscope and other basic electronic equipment to make measurements of the frequency and voltage response of some simple but important and useful circuits. The basic science is a small extension of your high school or university physics course when you covered resistors, capacitors, and inductors, along with Kirchoff's "loop" and "junction" rules. The coverage is brief, but you can go to any number of sources for more information, for example Chapter 3 in "Experiments in Modern Physics", Second Edition (2003) by Melissinos and Napolitano.

Lightning Review of Elementary Passive Circuits

Consider the following three representations of the same simple two-component circuit:



The drawing on the left is familiar to you, and you can easily show, using Kirchoff's loop rule, that the "voltage drop" across resistors R_1 and R_2 are $VR_1/(R_1 + R_2)$ and $VR_2/(R_1 + R_2)$, respectively. The middle figure just redraws this in a suggestive way. (The horizontal lines at the bottom are "ground", that is $V = 0$.) The black circle is a "tap" which is given by the "output" voltage $V_{\text{out}} = V_{\text{in}}R_2/(R_1 + R_2)$ for an input voltage $V = V_{\text{in}}$.

The drawing on the right is the same as the one in the middle, except that we talk about "impedance" Z instead of "resistance" R when the voltage is dependent on time, aka an "AC" voltage. This is important if the resistors are replaced by capacitors and inductors.

The relationship between voltage V and current I for a resistor is simply $V = IR$, so the impedance of a resistor is just $Z_R = R$. The charge Q on a capacitor C carrying a voltage V is $Q = CV$, so the current $I = dQ/dt = CdV/dt$. If $V(t) = V_0e^{i\omega t}$ (or, more properly, the real part of the right hand side) then

$$V = V_0e^{i\omega t} = IZ_C = CV_0(i\omega)e^{i\omega t}Z_C \qquad \text{so} \qquad Z_C = \frac{1}{i\omega C}$$

is the impedance of the capacitor.

The "i" in the denominator of Z_C is important! Since $i = e^{i\pi/2}$, it means that the current and voltage across a capacitor will be 90° out of phase with each other. In the measurements below, you will see how this works.

Inductors are similar. In this case, $V = LdI/dt$ for an inductor L, so write $I = I_0 e^{i\omega t}$ and

$$V(t) = L(i\omega)I_0 e^{i\omega t} = i\omega LI(t) \qquad \text{so} \qquad Z_L = i\omega L$$

Can you see why we say that a capacitor (inductor) acts like a short (open) circuit at high frequencies, and an open (short) circuit at low frequencies? (Think about what they are, physically, and the reason for this is obvious.) This is a handy way to quickly analyze how a simple passive circuit behaves.

Measurements

First, construct something that looks like the middle drawing with some resistors on a breadboard. Confirm that $V_{\rm out}$ is what you expect, both with a voltmeter (for a DC power supply) and with the oscilloscope (for an AC supply).

Next, construct a "low pass filter" with a resistor and capacitor. That is, you expect $V_{\rm out}$ to be zero for high frequencies, but $V_{\rm out} = V_{\rm in}$ for low frequencies. Derive an expression for both the magnitude and phase of $V_{\rm out}/V_{\rm in}$ as a function of frequency and identify the "cutoff" frequency in terms of R and C. Make measurements of $V_{\rm out}/V_{\rm in}$ as a function of frequency and compare to your predictions for magnitude and phase. (You will need to figure out what are the resistance and capacitance of your components and convert the units appropriately.)

Now make a "high pass filter" out of the same components and show that it also has the expected behavior. Explain why this is the case, comparing the behavior at high and low frequencies.

Then repeat either the "low pass" or "high pass" measurements with a resistor and an inductor. Show that the way you arrange the components is opposite from the way you did for a resistor and capacitor.

Finally, experiment with a circuit with all three components, namely one resistor, one capacitor, and one inductor. Arrange the components so that you have both a "notch" filter (where all frequencies are passed except for those within a certain range) and a "bandpass" filter (where only frequencies in a certain range are passed.) Derive the expected dependences of magnitude and phase of $V_{\rm out}/V_{\rm in}$ as a function of frequency and compare to your measurements.