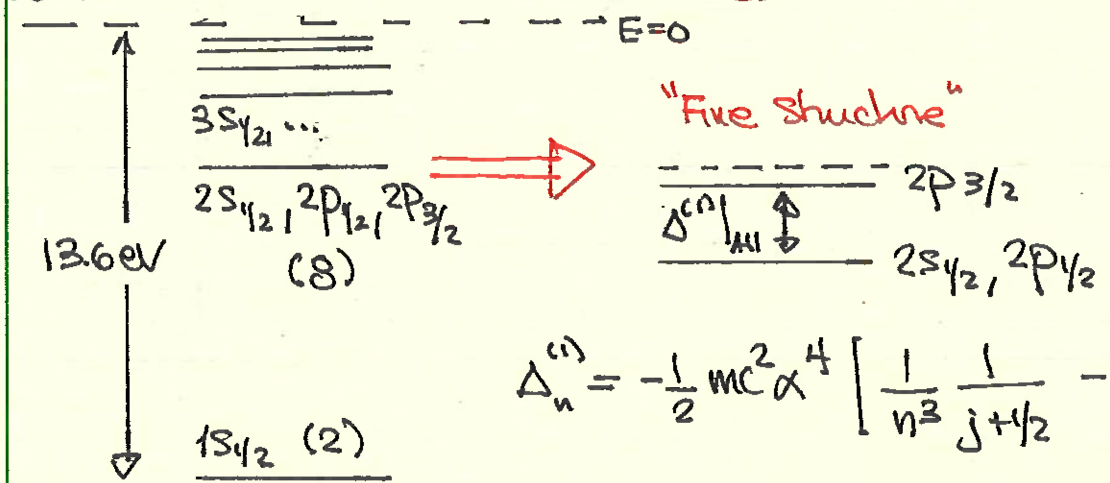
Phys 4702 Intro QM II Fall 2024 10 Oct 2024

* Class over Zoom from Boston
* Homework Status?

Review: Energy levels of Hydrogen



$$\Delta_n^{(1)} = -\frac{1}{2} mc^2 \alpha^4 \left[\frac{1}{n^3} \frac{1}{j+1/2} - \frac{3}{4n^4} \right]$$

Two choices for states $|n^{(0)}\rangle$ (For a given n)

$|\ell m_\ell s m_s\rangle = |\ell m_\ell ; \pm\hat{z}\rangle$ "L, S Basis"

$|\ell s j m\rangle$ "J Basis"

Must have $m_\ell + m_s = m = m_\ell \pm 1/2$

⇒ Only two states mix!

$$|j=\ell+1/2, m\rangle = \left[\frac{\ell+m_\ell+1}{2\ell+1}\right]^{1/2} |\ell, m_\ell ; +\hat{z}\rangle + \left[\frac{\ell-m_\ell}{2\ell+1}\right]^{1/2} |\ell, m_\ell+1 ; -\hat{z}\rangle$$

$$|j=\ell-1/2, m\rangle = \left[\frac{\ell-m_\ell}{2\ell+1}\right]^{1/2} |\ell, m_\ell ; +\hat{z}\rangle + \left[\frac{\ell+m_\ell+1}{2\ell+1}\right]^{1/2} |\ell, m_\ell+1 ; -\hat{z}\rangle$$

Today: Zeeman Effect (on Hydrogen)

[n.b. This is our last "hydrogen" class.]

Application of static magnetic field $\vec{B} = B\hat{z}$

⟺ Use first order perturbation theory

"Energy shifts" ⟹ "spectral lines split" (c(tw))

Two Effects:
- Electron spin ⟹ $\vec{\mu}\cdot\vec{B} \to \vec{S}\cdot\vec{B}$
- Electron orbital angular momentum ⟹ $\vec{L}\cdot\vec{B}$ (Recall HW#3 Prob. 3)

But Different "g-factors" ⟹ Not prop. to $\vec{J} = \vec{L} + \vec{S}$!!

⟺ Have to be mindful of degeneracies!

Finding the Perturbation V

- Electron spin: Easy!

$$V_S = -\vec{\mu}\cdot\vec{B} = -\left[-\frac{ge}{2mc}\vec{S}\right]\cdot B\hat{z} = \frac{g}{2}\frac{e}{mc}BS_z$$

We will just set $g = 2 \Rightarrow V_S = \frac{eB}{mc}S_z$

- Electron orbital angular momentum (Recall HW)

$\vec{B} = \nabla\times\vec{A}$ with $\vec{A} = -\frac{1}{2}By\,\hat{x} + \frac{1}{2}Bx\,\hat{y}$

and $H = \frac{1}{2m}\left(\vec{p} + \frac{e\vec{A}}{c}\right)^2$ [Electron charge $q = -e$]

Expand $\left(\vec{p}+\frac{e\vec{A}}{c}\right)^2 = \vec{p}^{\,2} + \frac{e}{c}(\vec{p}\cdot\vec{A}+\vec{A}\cdot\vec{p}) + \frac{e^2}{c^2}\vec{A}^{\,2}$

Standard canonical momentum

⇒ Part of "H_0"

"Quadratic Zeeman Effect"

Careful! $\vec{p}, \vec{A}$ Don't commute!

$$\vec{p}\cdot\vec{A}+\vec{A}\cdot\vec{p} = \underline{\vec{p}\cdot\vec{A}-\vec{A}\cdot\vec{p}} + 2\vec{A}\cdot\vec{p}$$

$$= [p_i, A_i] = -i\hbar\frac{\partial A_i}{\partial x_i} = -i\hbar\,\vec{\nabla}\cdot\vec{A} = 0$$

$$[\,\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z} = 0+0+0 \quad \text{for } \vec{A} = \frac{-B}{2}[y\hat{x}-x\hat{y}]\,]$$

⇒ "Linear" Zeeman perturbation is

$$V_L = \frac{1}{2m}\frac{e}{c}2\vec{A}\cdot\vec{p} = \frac{e}{2mc}B\,[-yp_x+xp_y] = \frac{eB}{2mc}L_z$$

i.e. $V_L = \frac{\text{"}g\text{"}}{2}\frac{e}{mc}B L_z$ with $\underline{\underline{g=1}}$ (not 2!!)

$$\Rightarrow\ V = V_L + V_S = \frac{eB}{2mc}(L_z + 2S_z)$$

$$= \frac{eB}{2mc}(J_z + S_z)$$

Diagonal in which basis??

Weak Field Zeeman Effect

i.e. Energy shift << Relativistic Corrections (K.E. + S.O.)

$\Rightarrow$ Consider effect on states $|n^{(0)}\rangle$ w/ definite j

[Energies just depend on n and j]

NOTE: $V = \dfrac{eB}{2mc}(J_z + S_z)$ commutes with $\vec{J}_z$ & $\vec{L}^2$

$\Rightarrow$ States w/ different m do not mix.

$[\vec{J}^2, V] \neq 0$ But Different j are Different states!

$\Rightarrow$ Calculate $\Delta^{(1)} = \langle jm | V | jm \rangle$

$$= \frac{eB}{2mc} \underbrace{\langle jm | J_z | jm \rangle}_{= m\hbar} + \frac{eB}{2mc} \langle jm | S_z | jm \rangle$$

Recall forms of $|j = \ell \pm 1/2, m\rangle$ in terms of $|\ell, m_\ell (+1); \pm\hat{z}\rangle$.

Also $m_\ell + m_s = m \Rightarrow m_\ell = m \mp \frac{1}{2}$ for $|\pm\hat{z}\rangle$

$$S_z |j = \ell + 1/2, m\rangle = \frac{\hbar}{2}\left[\frac{\ell + m + 1/2}{2\ell+1}\right]^{1/2} |\ell, m_\ell; +\hat{z}\rangle - \frac{\hbar}{2}\left[\frac{\ell - m + 1/2}{2\ell+1}\right]^{1/2} |\ell, m_\ell + 1; -\hat{z}\rangle$$

$$\Rightarrow \langle S_z \rangle_{j = \ell + 1/2} = \frac{\hbar}{2} \frac{1}{2\ell+1}\left[\ell + m + 1/2 - (\ell - m + 1/2)\right]$$

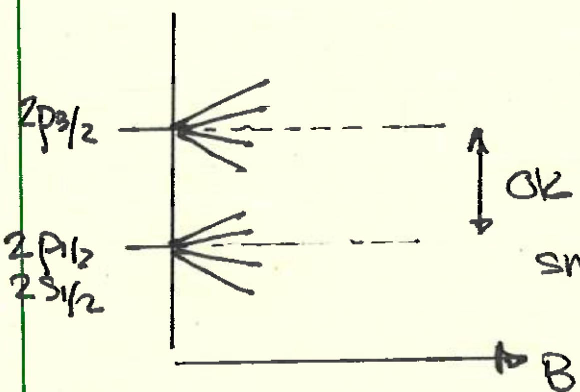
$$= + \frac{m\hbar}{2\ell+1} \qquad \text{Similarly } \langle S_z \rangle_{j = \ell - 1/2} = - \frac{m\hbar}{2\ell+1}$$

$$\Rightarrow \Delta^{(1)}_{j=\ell\pm 1/2} = \frac{eB}{2m_e c}\left[m\hbar \pm \frac{m\hbar}{2\ell+1}\right]$$

$$= \frac{e\hbar B}{2m_e c}\, m\, \underline{g(j=\ell\pm 1/2,\ell)}$$

$$g(j=\ell\pm 1/2,\ell) = 1 \pm \frac{1}{2\ell+1}$$ "Landé g-Factor"

<u>Homework</u>: Plot this.



OK until the shifts are not small compared to the gap

<u>Strong Field Zeeman Effect</u>

aka "Paschen-Back Effect"

Disregard spin-orbit splitting, or put it in later as a perturbation of the perturbation.

Write $V = \frac{eB}{2m_e c}(L_z + 2S_z)$

Commutes with $\vec{L}^2, L_z, \vec{S}^2, S_z$!

$\Rightarrow$ Use the $|n\ell m_\ell s m_s\rangle$ basis!

$$\Delta^{(1)} = \langle \ell m_\ell s m_s | V | \ell m_\ell s m_s \rangle$$

$$= \frac{eB}{2m_e c} \hbar \left(\underline{\underline{m_\ell + 2m_s}} \right)$$

Example: $n=2$ $\quad \ell=0 \quad m=0$
$\qquad\qquad\qquad \ell=1 \quad m=\pm1, 0$ $\Big\}$ $m_s = \pm\frac{1}{2}$

m_ℓ	$2m_s$	$m_\ell + 2m_s$
-1	-1	-2
0	-1	-1
$+1$	-1	0
-1	$+1$	0
0	$+1$	$+1$
$+1$	$+1$	$+2$

Splits into five "states"

NOTE: Can include spin-orbit effect using

$$\langle \ell m_\ell s m_s | \vec{L} \cdot \vec{S} | n \ell m_\ell s m_s \rangle$$

$$= \langle \ell m_\ell s m_s | \Big(L_z S_z + \frac{1}{2} \underbrace{[L_+ S_- + L_- S_+]}_{=0} \Big) | n \ell m_\ell s m_s \rangle$$

$$= m_\ell m_s$$

Homework: "the whole shebang"

Diagonalize V in the $|\ell s j m_j\rangle$ Basis for $n=2$.
"8×8 matrix" (But most elements $=0$)

$\Leftarrow\!\Rightarrow$ Watch as B increases from "weak" to "strong"