PHYS4702 Intro Quantum Mechanics II HW#12 Due 19 Nov 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) In class we wrote the Lorentz transformation matrix $a^\mu{}_\nu$ where $x'^\mu = a^\mu{}_\nu x^\nu$ for a boost of a vector x^μ along the $x = x^1$ axis.

(a) If $x'_\mu = a_\mu{}^\beta x_\beta$ describes the transformation for the covector, find the matrix $a_\mu{}^\beta$.

(b) Show that $a_\alpha{}^\mu a^\alpha{}_\nu = \delta^\mu_\nu$

(2) Prove that the differential operator $\partial_\mu \equiv \partial/\partial x^\mu = (\partial/\partial(ct), \vec{\nabla})$ (no "minus" signs) transforms like a covector. Now show that a function $A(x^\mu) = A(t, \vec{r})$ that satisfies $\partial_\mu \partial^\mu A = 0$ is a wave with speed c.

(3) Conservation of some conserved quantity, which we might as well call a "charge", requires

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

where $\rho(t, \vec{r})$ is the charge density and $\vec{j}(t, \vec{r})$ is the current density of the charge. (You've seen this equation, known as the "continuity equation," before.) Show that $j^\mu = (c\rho, \vec{j})$ transforms as a four-vector. (You'll need to remind yourself of the phenomenon known as "length contraction.") Then derive the covariant form of the continuity equation.

(4) Calculate values for the following quantities in the given natural units:

(a) The electron mass in MeV and the proton mass in GeV

(b) The Planck mass $m_p = (\hbar c/G)^{1/2}$ in GeV

(c) The momentum in MeV of a proton with kinetic energy 50 MeV

(d) The momentum in MeV of an electron with kinetic energy 50 MeV

(5) In the following, assume that $\psi(x^\mu)$ satisfies the Klein Gordon equation in natural units.

(a) Show that the following quantity represents a conserved current:

$$j^\mu \equiv \frac{i}{2m} \left[\psi^* \partial^\mu \psi - (\partial^\mu \psi)^* \psi\right]$$

(b) Write $\psi(t, \vec{r})$ as a plane wave in covariant form, and determine j^μ. Is the charge density $\rho = j^0$ a positive definite function?