# PHYS4702 Intro Quantum Mechanics II HW#11 Due 12 Nov 2024

*This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.*

**(1)** A two-state system, initially in state $|1\rangle$, is acted on by a Hamiltonian

$$H(t) \doteq \begin{bmatrix} E_1 & V_0 e^{-i\omega t} \\ V_0 e^{i\omega t} & E_2 \end{bmatrix} \doteq H_0 + V(t)$$

(a) Find the probability as a function of time that it is found in state $|2\rangle$ first by solving the problem exactly. (Recall Problem 4&5 in Homework #9.)

(b) Solve the problem using first order time-dependent perturbation theory, assuming $V(t)$ is "small." What has to be true for the two solutions to agree?

**(2)** In class, we made use of the relation

$$\lim_{\epsilon \to 0} \frac{\epsilon e^{2\epsilon t}}{\omega_{ni}^2 + \epsilon^2} = \pi\hbar\delta(E_n - E_i)$$

where $t$ is finite and $\hbar\omega_{ni} = E_n - E_i$. Prove this by showing that the limit gives zero ($\infty$) when $\omega_{ni} \neq 0$ ($= 0$) and that the integral gives the correct value.

**(3)** In class we derived the scattering Green's function

$$G(\vec{r}, \vec{r}') = -\frac{1}{4\pi} \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$$

Show that, for $\vec{r} \neq \vec{r}'$, $G(\vec{r}, \vec{r}')$ is the eigenfunction of the Hamiltonian for a free particle with mass $m$ and energy $E = \hbar^2 k^2/2m$.

**(4)** Use the Born approximation to calculate the differential cross section for a particle of mass $m$ scattering from a Yukawa potential, namely

$$V(r) = V_0 \frac{e^{-\mu r}}{\mu r}$$

Show that in the limit $\mu \to 0$ your answer agrees with the (classical) Rutherford scattering cross section.

**(5)** Use the Born approximation to calculate the differential cross section for a particle of mass $m$ scattering from a spherically symmetric potential energy function $V(r) = A/r^2$. Do you expect this to be even approximately equal to the classical cross section?