PHYS4702 Intro Quantum Mechanics II HW#10 Due 5 Nov 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Reproduce the argument we derived in class for the first order time dependent perturbation coefficient $c_n^{(1)}(t)$ in the interaction picture for a transition to the state $|n\rangle$ from an initial state $|i\rangle$, that is

$$c_n^{(1)}(t) = -\frac{i}{\hbar}\int_0^t e^{i\omega_{ni}t} V_{ni}(t')\, dt'$$

(2) A one-dimensional simple harmonic oscillator with frequency ω is in the ground state $|0\rangle$ at $t = 0$, when a space- and time-dependent perturbation given by

$$V(x,t) = V_0 \frac{2m\omega}{\hbar} x^2 e^{-t/\tau}$$

with constants V_0 and τ is turned on. Use time-dependent perturbation theory to find the probability that the oscillator makes a transition to the state $|n\rangle \neq |0\rangle$ when $t \gg \tau$.

(3) A one-dimensional simple harmonic oscillator with frequency ω_0 is in the ground state $|0\rangle$ at $t = 0$, when a space- and time-dependent perturbation given by

$$V(x,t) = Ax\cos\omega t$$

with constants V_0 and τ is turned on. Using first order perturbation theory, find the probability that the oscillator is in the $|n = 2\rangle$ state as a function of time for $t \geq 0$.

(4) Following what we did in class, derive an expression for the density of states for a free particle in three dimensions. Normalize the wave function in a box of side length L and apply periodic boundary conditions.

(5) Derive an expression for the density of states for a free particle in *two* dimensions. Normalize the wave function in a box of side length L and apply periodic boundary conditions. Compare to the result for three dimensions.