PHYS4702 Intro Quantum Mechanics II HW#9 Due 29 Oct 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) This problem is a review of concepts we have studied so far, including last semester.

(a) Find the matrix representations of S_x, S_y, and S_z in the $|s,m\rangle$ basis for $s=1$.

(b) A spin-1 particle with magnetic moment $\vec{\mu} = -\gamma\vec{S}$ is placed in a static magnetic field $\vec{B} = B\hat{\mathbf{z}}$. The particle is in the $+\hbar$ eigenstate of S_y at $t=0$. Find the normalized state vector as a function of time.

(c) Calculate $\langle S_x\rangle$, $\langle S_y\rangle$, $\langle S_z\rangle$ as a function of time. Explain why this what you expect.

(2) You've heard of the "energy-time" uncertainty principle $\Delta E\Delta t \geq \hbar/2$, but time is not an observable, so this doesn't really make any sense. Show, however, that if you define Δt in terms of some time-independent observable Q as $\Delta t = \Delta Q/|d\langle Q\rangle/dt|$, then you can derive this relationship. Why is this a reasonable definition of Δt?

(3) A tritium atom is a hydrogen atom where the nucleus ^{3}H is one proton bound to two neutrons. The nucleus is radioactive and decays to a ^{3}He nucleus, while emitting an electron with about 10 keV kinetic energy.

(a) Compare the time it takes the electron to leave the atom with the inverse of the frequency between the ground and first excited state of the ^{3}He ion. Use this to justify the use of the "sudden approximation."

(b) Calculate the probability that the ^{3}He ion ends up in the electronic ground state.

You might be curious to look up something called the KATRIN experiment.

(4) & (5) Magnetic resonance is induced by the Hamiltonian $H = -\vec{\mu}\cdot\vec{B}$ where

$$\vec{B} = B_0\hat{\mathbf{z}} + B_1(\cos\omega t\,\hat{\mathbf{x}} + \sin\omega t\,\hat{\mathbf{y}})$$

is a magnetic field consisting of a static component with magnitude B_0 in the z-direction, and a rotating component with magnitude B_1 in the xy-plane. Write $\vec{\mu} = -\gamma\vec{S}$ for spin operator $\vec{S}$. Assume a spin-1/2 system and use the $|\pm\hat{\mathbf{z}}\rangle$ basis.

(a) For the interaction picture state $|\alpha;t\rangle_I = c_1(t)\,|+\hat{\mathbf{z}}\rangle + c_2(t)\,|-\hat{\mathbf{z}}\rangle$, find the two coupled differential equations for $c_1(t)$ and $c_2(t)$ in terms of $\omega_0 \equiv \gamma B_0$ and $\omega_1 \equiv \gamma B_1$.

(b) Assuming the initial state is $|\alpha\rangle = |+\hat{\mathbf{z}}\rangle$, derive Rabi's Formula for the transition probability for magnetic resonance, namely

$$|\langle -\hat{\mathbf{z}}|\alpha;t\rangle|^2 = \frac{\omega_1^2}{(\omega_0-\omega)^2+\omega_1^2}\sin^2\left(\frac{\sqrt{(\omega_0-\omega)^2+\omega_1^2}}{2}\,t\right)$$

(c) Make a plot of the maximum transition probability versus ω for $B_0/B_1 = 1000$.