PHYS4702 Intro Quantum Mechanics II HW#7 Due 15 Oct 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) An analysis of the Dirac equation leads to the perturbation known as the Darwin term

$$V_D = -\frac{1}{8m^2c^2}\sum_{i=1}^{3}[p_i,[p_i,V(r)]]$$

where $V(r)$ is the electrostatic potential energy of the nucleus. Show that for a hydrogen atom, this leads to an energy shift that is exactly equal to the spurious $l = 0$ term from the spin-orbit perturbation.

(2) You have diagonalized $2\vec{L}\cdot\vec{S}$ to write the $|lsjm\rangle$ in terms of the $|lm_lsm_s\rangle$ basis. Now use the $|lsjm\rangle$ basis to find the first order energy shifts for all eight $n = 2$ levels in hydrogen from the spin-orbit interaction *plus* an arbitrarily strong magnetic field $\vec{B} = B\hat{\mathbf{z}}$. This requires you to diagonalize an 8×8 matrix with two terms in the perturbation, but it is easier than you think. Make a plot of the perturbed energies as a function of B and show that the eight states at low B merge into the five states at high B as discussed in class. (You can just parameterize the constant factors in the two terms of the perturbation and set them equal to some convenient values to make the plot.)

(3) The hydrogen atom is diamagnetic, which means that an applied magnetic field induces a magnetic dipole moment in the opposite direction. This is caused by the square of the vector potential $\vec{A}$, which we neglected when calculating the Zeeman effect. It can be observed because it induces an energy shift in the ground state that is quadratic in the field strength

$$\Delta_{1s} = -\frac{1}{2}\chi\vec{B}^2$$

where χ is called the magnetic susceptibility. Find an expression for χ using first order perturbation theory. You can make use of your result from Problem 3 on Homework #3.

(4) Make and label a diagram of the energy levels of hydrogen, including specific energies (in eV) of the ground, first, and second excited states, in zeroth order. Then make and label diagrams of the $n = 2$ and $n = 3$ energy levels including all first order relativistic corrections.

(5) For some of the following problems, be aware that we use CGS units in class, but most textbooks on electromagnetism use SI units.

(a) Use your result from **(4)** above to calculate the wavelength of the $n = 3 \to n = 2$ transition in hydrogen. What color is this "line"?

(b) What is the line splitting $\Delta\lambda$ between the $3p_{3/2} \to 2s_{1/2}$ and $3p_{1/2} \to 2s_{1/2}$ transitions?

(c) How strong an electric field is needed to induce a Stark effect splitting of the $n = 2$ states of hydrogen that is the same as the energy difference between the $j = 1/2$ and $j = 3/2$ states for $n = 2$ from relativistic corrections?

(d) Determine (in Gauss) the size of the applied magnetic fields that would qualify as "weak" or "strong" with regard to the Zeeman splitting of the $n = 2$ states in hydrogen?