PHYS4702 Intro Quantum Mechanics II HW#5 Due 1 Oct 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) In class, we calculated the second order energy shift for a perturbation $v(x) = \varepsilon x$ to a simple harmonic oscillator with potential $V(x) = m\omega^2 x^2/2$. Now calculate the first order correction $|0^{(1)}\rangle$ to the ground state ket $|0\rangle$ and use it to determine the ground state expectation value $\langle x\rangle$. Show that your result agrees with the exact calculation to first order.

(2) The matrix representation of some Hamiltonian H is

$$H \doteq \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} + \begin{bmatrix} 0 & V & 0 \\ V & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Assuming that E_1, E_2, and E_3 are all positive and that $V \ll E_i$ for all E_i,

(a) Calculate the exact energy eigenvalues and eigenvectors, and expand to second order in V. What other tacit assumption do you need to make?
(b) Find the eigenvalues through second order and their eigenvectors to first order using perturbation theory with $E_1 > E_2 \neq E_3$ and $E_1 \neq E_3$. Compare to the exact values.
(c) Repeat for $E_1 = E_2 \equiv E_0 \neq E_3$. (Just find the eigenvalues, not the eigenvectors.)

(3) A mass m is attached to the end of a massless rigid rod of length a. The other end of the rod is fixed at the origin. The rod rotates in a plane about the origin.

(a) Find the Hamiltonian and solve the Schrödinger equation in plane polar coordinates (ρ, ϕ) to find the normalized wave functions and their energy eigenvalues.
(b) Add a perturbation $V(\phi) = V_0 \cos(2\phi)$ and calculate the first order energy shifts in the three lowest energy levels. Be mindful of degeneracies.
(c) Calculate the second order energy shift of the ground state.

(4) A two-dimensional isotropic harmonic oscillator has potential energy $m\omega^2(x^2 + y^2)/2$.

(a) Find the energies and degeneracies of the three <u>states</u> with the lowest energies.
(b) Apply a perturbation $V(x, y) = \varepsilon m\omega^2 xy$, where $\varepsilon \ll 1$, and calculate the first order energy shifts to the three states in (a).
(c) Solve the problem exactly and compare to the results obtained above.

(5) *This problem is based on material from last semester, and is a "warm up exercise" for what we'll cover next week.* An atom in an electron has orbital angular momentum $\vec{L}$ and spin angular momentum $\vec{S}$, which couple to form $\vec{J} = \vec{L} + \vec{S}$.

(a) Show that $\vec{J}$ obeys the correct angular momentum commutation relations.
(b) Show that $\vec{J}^2$ commutes with $\vec{L}^2$ and $\vec{S}^2$.
(c) Show that $\vec{J}^2$ does not commute with L_z or S_z.