PHYS4702 Intro Quantum Mechanics II HW#4 Due 24 Sep 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Consider a one-dimensional infinite square well with $-a \leq x \leq a$. Calculate the first order shifts for the first *three* energy levels for the following perturbations:

(a) $V(x) = V_0 x/a$
(b) $V(x) = V_0(x+a)/a$
(c) $V(x) = V_0(a-x)/a$ for $x \geq 0$ and $V(x) = V_0(x+a)/a$ for $x \leq 0$.

Explain the obvious qualitative differences between your three results. You might find it helpful to plot the three potentials on the same set of axes.

(2) Consider a particle of mass m moving in a one-dimensional harmonic oscillator potential $V(x) = m\omega^2 x^2/2$. Add a "perturbation" $v(x) = m\varepsilon x^2/2$ and calculate the energy shift of the ground state through second order. (That is, calculate the first and second order energy shifts.) Compare your result to the exact answer.

(3) Calculate the first order energy shift for the four $n = 2$ states of the hydrogen atom from taking into account the finite size of the proton. (You can assume the proton is a uniformly charged sphere of radius R.) Write your results to lowest nonzero order in R/a_0 where a_0 is the Bohr radius, and show that that these energy shifts are small changes to the $n = 2$ energy eigenvalue. Also explain why your result for one of the states is much larger than what you get for the other three. (I suggest using MATHEMATICA to calculate the wave functions and do the integrals. The function Series is handy for doing a lowest order expansion in terms of $x \equiv R/a_0$.)

(4) Consider a particle of mass m moving in a one-dimensional harmonic oscillator potential $V(x) = m\omega^2 x^2/2$. Derive a lowest order relativistic perturbation by expanding the kinetic energy in momentum and calculate the first order energy shift in the ground state. You can consider doing this by writing the momentum in terms of creation and annihilation operators, or by looking up the ground state wave function and calculating the energy shift in terms of wave mechanics.

(5) This problem is about an approximation technique called the *variational principle.*

(a) The ground state for some Hamiltonian H is $|\Psi\rangle$ with $H|\Psi\rangle = E_0|\Psi\rangle$. If $|\Phi\rangle$ is any other state, and $\bar{H} \equiv \langle\Phi|H|\Phi\rangle/\langle\Phi|\Phi\rangle$, prove that $\bar{H} \geq E_0$.
(b) For the one-dimensional infinite square well with $-a \leq x \leq a$ and $\langle x|\Phi\rangle = N(a^2 - x^2)$, calculate $\bar{H}$ and compare to E_0. Plot the (normalized) trial wave function on top of the true wave function.
(c) Repeat for $\langle x|\Phi\rangle = N(a^\alpha - |x|^\alpha)$ with finding the value of α that minimizes $\bar{H}$. Compare the accuracy of this trial wave function to that in (b).