PHYS4702 Intro Quantum Mechanics II HW#3 Due 17 Sep 2024

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Two identical particles of mass m move horizontally with the same momentum over some time T. One of the particles is higher than the other by a distance L. Solve the Schrödinger equation to show that the upper particle accumulates a phase difference of $\Delta\phi = -mgLT/\hbar$ relative to the lower one. Show that this effect for thermal neutrons (energy close to kT at room temperature) leads to an easily observable number of interference fringes for distances of 5 cm. You might want to look up the paper by Colella, et al, Phys.Rev.Lett. 34(1975)1472.

(2) For a particle with mass m and charge q, the kinematical momentum $\vec{\Pi}$ is defined as

$$\vec{\Pi} = \frac{d\vec{r}}{dt} = \vec{p} - \frac{q}{c}\vec{A}$$

where $\vec{A}$ is the vector potential and $\vec{p}$ is the usual canonical momentum that generates translations. Show that

$$[\Pi_i, \Pi_k] = i\hbar\frac{q}{c}\epsilon_{ijk}B_k$$

(3) Consider a particle with mass m and charge q in a static magnetic field $\vec{B} = B\hat{z}$. Find an expression for $\vec{A}$ with $\vec{\nabla}\cdot\vec{A} = 0$, and write the Hamiltonian in terms of $\vec{p}$ and $\vec{A}$, expanding out the kinetic energy in Cartesian coordinates x and y. Show that your result includes the expected interaction between the orbital magnetic moment $(q/2mc)\vec{L}$ with $\vec{B}$. You should also find an additional term; comment on its physical significance.

(4) Consider a particle with mass m and charge q in a static magnetic field $\vec{B} = B\hat{z}$. Compare the commutation relation $[\Pi_x, \Pi_y]$ from Problem (2) with those for the one-dimensional simple harmonic oscillator to show that the energy eigenvalues are given by

$$E_{k,n} = \frac{\hbar^2 k^2}{2m} + \frac{|qB|\hbar}{mc}\left(n + \frac{1}{2}\right)$$

where the eigenvalues of p_z are $\hbar k$ and $n = 0, 1, 2, \ldots$. Note that this solution *does not* involve anything like a lengthy calculation!

(5) An electron is confined inside a cylindrical shell $\rho_a \le \rho \le \rho_b$ around the z-axis, and $0 \le z \le L$. Solve the Schrödinger equation in cylindrical coordinates to find energy eigenvalues

$$E_{lmn} = \left(\frac{\hbar^2}{2m_e}\right)\left[k_{mn}^2 + \left(\frac{l\pi}{L}\right)^2\right] \quad (l = 1, 2, 3, \ldots, m = 0, 1, 2, \ldots),$$

where k_{mn} is the nth root of $J_m(k_{mn}\rho_b)N_m(k_{mn}\rho_a) - N_m(k_{mn}\rho_b)J_m(k_{mn}\rho_a) = 0$, and $J_\ell(\rho)$ and $N_\ell(\rho)$ are linearly independent solutions to Bessel's equation. Repeat for a uniform magnetic field $\vec{B} = B\hat{z}$ only for $\rho < \rho_a$. If we require the ground-state energy to be unchanged in the presence of $\vec{B}$, show that we obtain "flux quantization"

$$\pi\rho_a^2 B = \frac{2\pi n\hbar c}{e} \quad \text{where} \quad (n = 0, \pm 1, \pm 2, \ldots)$$