

## The Thomas precession gives $g_e - 1$ , not $g_e / 2$

Roger R. Haar and Lorenzo J. Curtis

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## Comment on the buoyant nonbouncer

J. R. Torczynski

Fluid and Thermal Sciences Department, Sandia National Laboratories, Albuquerque, New Mexico 87185

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Recently I. R. Lapidus put forth the buoyant bouncer<sup>1</sup> problem. He proposed dropping a ball of radius  $R$  and density  $\rho$  into water (density  $\rho_w > \rho$ ) from a height  $H$  and presented a calculation of the time required for the ball to return to its original height. Lapidus made two assumptions: Friction losses are negligible, and the ball radius is negligible compared to the height ( $R \ll H$ ). These assumptions are contradictory under any reasonable conditions.

When the ball has reached the water surface, it will be traveling with a speed  $U = (2gH)^{1/2}$ . On entering the water, the ball will experience a drag force<sup>2</sup>

$$F_D \equiv \frac{1}{2} \rho_w U^2 C_D \pi R^2 = \rho_w g H C_D \pi R^2,$$

where  $C_D$  is the coefficient of drag. The coefficient of drag  $C_D$  is a function of the Reynolds number  $Re = UD/\nu$ , where  $D = 2R$  and  $\nu$  is the kinematic viscosity (for water,  $\nu \approx 10^{-6} \text{ m}^2/\text{s}$ ). For Reynolds numbers less than  $3 \times 10^5$ , it is found that  $C_D > 0.5$ ; in fact, for low Reynolds numbers (low velocities)  $C_D \approx 24/Re \rightarrow \infty$  as  $Re \rightarrow 0$  (Stokes' formula). For larger Reynolds numbers,  $C_D$  does dip to around 0.2 but apparently doesn't vanish (see Landau and Lifshitz<sup>2</sup> for a graph of  $C_D$  as a function of  $Re$ ).

To satisfy the requirement of negligible friction losses, we require that  $F_D \ll F_G$ , where we take the gravitational/buoyancy force<sup>3</sup>  $F_G$  as given by Lapidus,

$$F_G = mg \left( \frac{\rho_w - \rho}{\rho} \right) = \frac{4\pi}{3} R^3 g (\rho_w - \rho).$$

When the algebra is simplified, we are left with the following relation:

$$H \ll \frac{4}{3C_D} \left( 1 - \frac{\rho}{\rho_w} \right) R.$$

If with the symbol " $\ll$ " we are denoting at least one order of magnitude (factor of 10), then combining this relation with the other assumption of Lapidus ( $R \ll H$ ) yields the following requirement:

$$C_D \ll \frac{4}{300} \left( 1 - \frac{\rho}{\rho_w} \right).$$

Even under the most favorable conditions, it is impossible to satisfy this requirement.

The reason this conflict arises is that the physics of Lapidus' assumptions incorrectly describes the real situation. The motion of a sphere in water will be drag dominated at these gravitationally induced speeds: for  $R \ll H$ , the impact velocity of the sphere will be far greater than its terminal velocity. (There is also the strong likelihood that surface waves, gravity and/or capillary, will be very important energy sinks.) At this point a statement by Richard Feynman<sup>5</sup> is particularly appropriate:

John von Neumann was well aware of the tremendous difference between what happens when you don't have the viscous terms and when you do, and he was also aware that, during most of the development of hydrodynamics until about 1900, almost the main interest was in solving beautiful *mathematical* problems with this approximation which had almost nothing to do with real fluids. He characterized the theorist who made such analyses as a man who studied "dry water." Such analyses leave out an *essential* property of the fluid.

<sup>1</sup>I. R. Lapidus, Am. J. Phys. **54**, 10 (1986).

<sup>2</sup>L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1959), pp. 168–172.

<sup>3</sup>It should be observed that these forces accelerate the surrounding water as well as the ball, so the expression for the *acceleration* of the ball must also include the *apparent mass* of water that is accelerated, as Crawford<sup>4</sup> notes.

<sup>4</sup>F. S. Crawford, Am. J. Phys. **54**, 584 (1986).

<sup>5</sup>R. P. Feynman, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1965), Vol. II, p. 40-3.

## The Thomas precession gives $g_e - 1$ , not $g_e/2$

Roger R. Haar and Lorenzo J. Curtis

Department of Physics and Astronomy, University of Toledo, Toledo, Ohio 43606

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Modern experimental techniques such as laser-rf excitation, quantum beat spectroscopy, level crossing measurements, etc. provide direct determinations of atomic fine structure splittings, often to uncertainties of less than

$10^{-10}$  Rydberg units. For hydrogenlike atoms these splittings can be predicted by a neat and concise semiclassical development of the spin-orbit interaction energy, presented in many textbooks on atomic physics (Refs. 1–5), that is

based on the Biot–Savart law and the vector model of angular momentum. Although the data available are of sufficient accuracy that self-energy corrections to the anomalous magnetic moment of the electron are necessary to obtain agreement, these contributions are neglected in most textbook developments. The purpose of this note is to indicate how self-energy corrections can be retained in the standard exposition of the spin-orbit energy with no increase in complexity, producing an expression that agrees with the experiment to within four parts in  $10^5$ .

The standard textbook development<sup>1–5</sup> of the spin-orbit interaction energy considers the magnetic field seen by an electron in a hydrogenlike atom due to the apparent motion of the nucleus, which is given by (in standard symbols<sup>1–8</sup>)

$$\mathbf{B} = \frac{k}{c^2} \frac{Ze(\mathbf{r} \times \mathbf{v})}{r^3} = \frac{Zke}{mc^2} \frac{\mathbf{L}}{r^3}, \quad (1)$$

and the anomalous magnetic moment of the electron, given by

$$\boldsymbol{\mu}_s = -g_e \frac{e}{2m} \mathbf{S}. \quad (2)$$

In the rest frame of the electron the interaction energy for these two charge circulations is

$$\Delta E = -\boldsymbol{\mu}_s \cdot \mathbf{B} = g_e \frac{Zke^2}{2(mc)^2} \frac{\mathbf{L} \cdot \mathbf{S}}{r^3}. \quad (3)$$

To transform to the rest frame of the nucleus it is necessary to take into account the Thomas precession. This is discussed in Refs. 1, 6, and 7, and requires that the Larmor frequency due to electron spin be corrected by the addition of the transformational Thomas frequency. The Thomas precession is in a direction opposite to the electron Larmor precession, and involves all of the same quantities except for the electron  $g$  factor. The frequency after transformation to the rest system of the nucleus is

$$\boldsymbol{\omega} = g_e \frac{e\mathbf{B}}{2m} - \frac{e\mathbf{B}}{2m} = (g_e - 1) \frac{e\mathbf{B}}{2m}. \quad (4)$$

Since the energy is given by

$$\Delta E = \boldsymbol{\omega} \cdot \mathbf{S} = (g_e - 1) \frac{Zke^2}{2(mc)^2} \frac{\mathbf{L} \cdot \mathbf{S}}{r^3}, \quad (5)$$

the  $g_e - 1$  factor also occurs in the energy. At this point most textbooks approximate the anomalous magnetic moment by the Dirac moment  $g_e(D) = 2$ , neglecting the electron self-energy corrections that are given by

$$g_e = 2 + (\alpha/\pi) - 0.657(\alpha/\pi)^2 + \dots. \quad (6)$$

(Models for the origin of these corrections have been presented by Grotch and Kazes.<sup>8</sup>) Some textbook developments substitute  $g_e(D) - 1 = 1$ , removing the explicit dependence upon  $g_e$ , whereas others substitute  $g_e(D) - 1 = g_e(D)/2$ . Although the use of the multiplicative factor  $g_e/2$  to make the additive correction for the Thomas precession is technically correct in the limit  $g_e \rightarrow g_e(D) = 2$ , its use can be misleading to students, since

$$g_e/2 = 1 + \alpha/2\pi + \dots, \quad (7)$$

whereas

$$g_e - 1 = 1 + \alpha/\pi + \dots. \quad (8)$$

Possible confusion in comparing textbook expressions describing atomic physics and quantum electrodynamics could be avoided simply by retaining the general correction factor  $g_e - 1$  in the expression for the fine structure. The self-energy corrections are important, amounting to 0.23%, substantially larger than, e.g., reduced mass corrections that are only 0.05%.

If self-energy corrections are included through the  $g_e - 1$  factor, and the expectation values

$$\langle \mathbf{L} \cdot \mathbf{S} \rangle = [j(j+1) - l(l+1) - s(s+1)] \hbar^2/2 \quad (9)$$

and

$$\langle r^{-3} \rangle = (Z/a_0 n)^3 [l(l + \frac{1}{2})(l+1)]^{-1} \quad (10)$$

are substituted into Eq. (5) and reduced mass corrections are made, the expression for the fine structure separation between two levels with the same  $n, l, s$ , and  $j = l + s(+)$  and  $j = l - s(-)$  obtained from Eqs. (5), (8)–(10) is

$$\langle \Delta E(+ ) - \Delta E(- ) \rangle = \frac{(1 + \alpha/\pi)}{(1 + m_e/M_p)} \frac{R\alpha^2 Z^4}{n^3 l(l+1)} (2s). \quad (11)$$

For the fine structure of the  $2p$  term in hydrogen, Eq. (11) yields 10 968.74 mHz, which is within four parts in  $10^5$  of the experimental value of 10 969.127(87) mHz.<sup>10</sup>

In addition to its application to hydrogenlike values, Eq. (11) is also useful in describing the fine structure of x ray and optical spectra in complex many-electron atoms, through the use of a noninteger screened charge  $Z \rightarrow Z - S$ . Eq. (11) is the lowest-order term in an  $\alpha Z$  expansion of the Dirac energy, and an explicit general expression for generating higher order corrections is given in Ref. 11.

Thus, within the spirit of the heuristic derivation, this exposition of the spin-orbit interaction accounts for the self-energy corrections through the factor  $g_e - 1$ , and provides a useful pedagogic example which illustrates the very high precision that is possible in atomic spectroscopy.

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<sup>1</sup>R. Eisberg and R. Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (Wiley, New York, 1985), 2nd ed., p. 279 and Appendix O.

<sup>2</sup>B. W. Shore and D. H. Menzel, *Principles of Atomic Spectra* (Wiley, New York, 1968), p. 311.

<sup>3</sup>N. Tralli and F. R. Pomilli, *Atomic Theory, An Introduction to Wave Mechanics* (McGraw-Hill, New York, 1969), p. 170.

<sup>4</sup>J. Norwood, Jr., *Twentieth Century Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1976), p. 236.

<sup>5</sup>S. Wieder, *The Foundations of Quantum Theory* (Academic, New York, 1973), p. 178.

<sup>6</sup>G. P. Fisher, *Am. J. Phys.* **39**, 1528 (1971); **40**, 1772 (1972).

<sup>7</sup>V. V. Dixit and L. P. S. Singh, *Am. J. Phys.* **48**, 415 (1980); **49**, 283 (1981).

<sup>8</sup>H. Grotch and E. Kazes, *Am. J. Phys.* **45**, 618 (1977).

<sup>9</sup>M. W. P. Strandberg, *Am. J. Phys.* **54**, 321 (1986).

<sup>10</sup>E. R. Cohen and B. N. Taylor, *J. Phys. Chem. Ref. Data* **2**, 663 (1973).

<sup>11</sup>L. J. Curtis, *J. Phys. B* **10**, L641 (1977).