PHYS4702 Atomic, Nuclear, & Particle Physics Fall 2015 HW #11

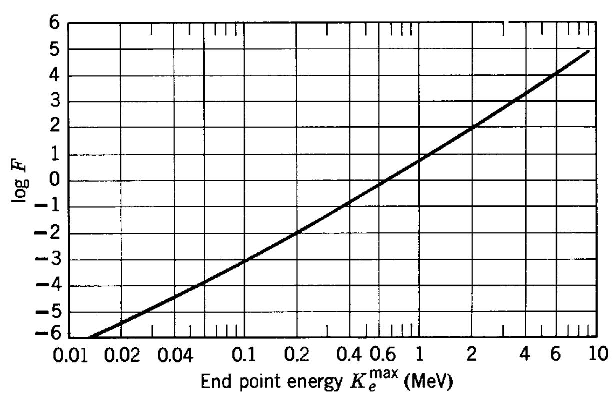
Due at the start of class on Thursday 12 Nov 2015

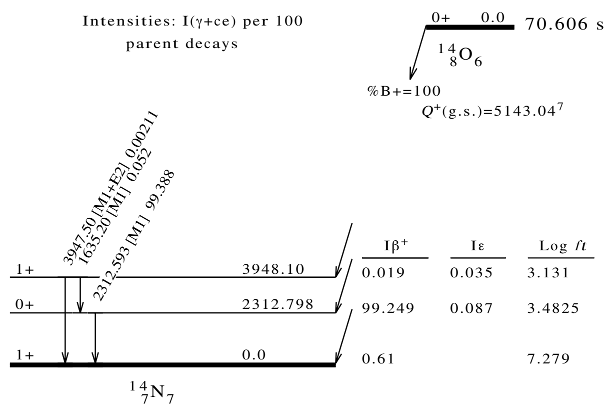
(1) Consider the three-atom decay sequence $A \to B \to C$, where C is stable. Defining $\lambda = 1/T$ where T is the mean life, and starting with a pure sample of $N_A^{(0)}$ atoms A, show that the number of atoms B at a time t is given by

$$N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A^{(0)} \left[e^{-\lambda_A t} - e^{-\lambda_B t} \right]$$

Take each of the three limits $t \ll \{\lambda_A, \lambda_B\}$, $T_A \ll t \ll T_B$ (for $T_A \ll T_B$), and $T_B \ll t \ll T_A$ (for $T_B \ll T_A$), and explain why these are the expected results for $N_B(t)$.

(2) The graph below on the left, figure16-13 from your textbook, plots the function $F(K_e^{\max})$, that is the integral of the beta decay phase space function, in a low Z nucleus for which the Coulomb correction can be neglected. On the right is shown the beta decay information for ${}^{14}\text{O} \to {}^{14}\text{N} + e^+ + \bar{\nu}_e$, taken from http://www.nndc.bnl.gov/ensdf/.





Using Mathematica or some other program, reproduce Fig.16-13. You'll need to first deduce the factors and integral for the dimensionless function $F(K_e^{\max})$. Compare (16-12) and (16-17) but be careful of the words before (16-12); a more mathematically consistent way to write the left hand side of that equation would be dR/dp_e.

Then use this to calculate the three $\log(FT)$ values shown on the level diagram of ${}^{14}\text{O}$ decay. (The relevant data is included in the level diagram.) Note that the half-life of ${}^{14}\text{O}$ is given in seconds, and branching fraction to individual final states is given by $I\beta^+$ in percent.

(3) The following table lists some measurements of a beta spectrum $R(p_e)$ as a function of electron momentum p_e:

p_e (MeV/c)	1.0	1.25	1.5	1.75	2.0	2.25	2.5
R (counts)	4500	5750	6310	6380	5670	4630	3100

Make a Kurie plot using this data, and determine the decay energy E.