PHYS4702 Atomic, Nuclear, & Particle Physics Fall 2015 HW #6

Due at the start of class on Thursday 1 Oct 2015

(1) Determine the field gradient of a 1 m long Stern-Gerlach magnet that would produce a separation of 2 mm at the end of the magnet between the two components of a beam of silver atoms emitted with typical kinetic energy from a 2200°C oven. The magnetic dipole moment of silver is due to a single $\ell = 0$ electron, just as for hydrogen.

(2) Prove that the only possible values of the quantum number j from the series

$$j = \ell + \frac{1}{2}, \ell - \frac{1}{2}, \ell - \frac{3}{2}, \ldots$$

that satisfy the inequality

$$\sqrt{j(j+1)} \geq \left| \sqrt{\ell(\ell+1)} - \sqrt{s(s+1)} \right|$$

with $s = 1/2$ are

$$j = \ell \pm \frac{1}{2} \text{ if } \ell \neq 0 \qquad \text{or} \qquad j = \frac{1}{2} \text{ if } \ell = 0$$

(3) Consider all of the states of the hydrogen atom for $n = 3$, including the electron spin degree of freedom. By taking into account all possible values of ℓ, m_ℓ, s, and m_s, count the total number of states. Then, classify the states according to ℓ and j, using standard spectroscopic notation, and count up the total (including the m_j degeneracy) to show that you get the same total number. Indicate the relative energies of the states including spin-orbit splitting, and check your answer against some figure you are able to find on the web.