PHYS4702 Atomic, Nuclear, & Particle Physics Fall 2015 HW #2

Due at the start of class on Thursday 3 Sept 2015

(1) The electric field from a point charge, or the gravitational field from a point mass, has the form

$$\mathbf{V}(\mathbf{r}) = \frac{k}{r^2}\hat{\mathbf{r}} = \frac{k}{r^3}\mathbf{r}$$

where $\hat{\mathbf{r}}$ is a unit vector in the direction of the position vector $\mathbf{r}$, and k is a constant. Show by explicit differentiation in cartesian coordinates, that $\boldsymbol{\nabla} \cdot \mathbf{V} = 0$ everywhere except the origin. Then show that for a spherical surface around the origin, $\oint \mathbf{V} \cdot d\mathbf{A} = 4\pi k$. Hence, show that you can derive the integral form of Gauss' Law for a point charge.

(2) Estimate the rough order of magnitude of the length of time that an ice pick can be balanced on its point if the only limitation is that set by the Heisenberg uncertainty principle. Assume that the point is sharp and that the point and the surface on which it rests are hard. You may make approximations which do not alter the general order of magnitude of the result. Assume reasonable values for the dimensions and weight of the ice pick. Obtain an approximate numerical result and express it *in seconds*.

By the way, this is a photo of a typical ice pick, about eight inches long:

(3) The wave function for a particle of mass m in a square well over $-a \leq x \leq a$ with infinite sides, is given by $\psi(x) = A\sin(\pi x/a)$, where A is a constant. Substitute $\psi(x)$ into the time-independent Schrödinger equation to obtain the energy eigenvalue. Find the value of A that properly normalizes $\psi(x)$, and determine the expectation values $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$. Compare the resulting $\Delta x \Delta p$ to the Uncertainty Principle.