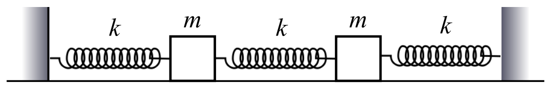
PHYS4702 Atomic, Nuclear, & Particle Physics Fall 2015 HW #1

Due at the start of class on Thursday 27 Aug 2015

(1) Two equal masses m move in one dimension and are each connected to fixed walls by springs with stiffness k. The masses are also connected to each other by a third, identical spring, as shown:



Write the (differential) equations of motion for the positions $x_1(t)$ and $x_2(t)$ of the two masses. (You are welcome to use Lagrangian formalism or just plane "$F = ma$.") Solve those equations with the ansatz $x_1(t) = A_1 e^{i\omega t}$ and $x_2(t) = A_2 e^{i\omega t}$; you will discover nontrivial solutions only for two values of ω^2. (Those two values are called *eigenfrequencies.*) What kind of motion corresponds to each of these two eigenfrequencies?

(2) A function $f(x)$ is periodic, such that $f(x+2) = f(x)$. For $-1 < x < 0$ $f(x) = -1$, and for $0 < x < 1$ $f(x) = +1$. Find the first five terms of the Fourier expansion for $f(x)$, and make a plot of the approximations based on the first term, and the sums up to the third and fifth terms, along with a plot of $f(x)$ itself.

(3) Prove the principle of linear superposition for the classical wave equation in one spatial dimension, namely

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} - \frac{\partial^2 y}{\partial x^2} = 0$$

That is, show that if $y_1(x,t)$ and $y_2(x,t)$ are solutions of the wave equation, then $y(x,t) = ay_1(x,t) + by_2(x,t)$ is also a solution, where a and b are arbitrary constants.