PHYS4101 Thermal Physics
(Fall 2021)

Midterm Exam #1
Tuesday 28 Sep 2021

There are **four questions** and you are to work all of them. You are welcome to use your textbook, notes, or any other resources, but you may not communicate with another human. Of course, if you have questions, you are encouraged to ask the person proctoring the exam.

The four problems will be equally weighted but *they are not of equal difficulty.* If you are stuck on one, move on to another and come back if you have time.

Please start each problem on a new page in your exam booklet.

Good luck!

(1) A monatomic ideal gas containing N atoms undergoes an adiabatic (i.e. $Q = 0$) compression from an initial volume $V_i = V$ to a final volume $V_f = V/2$. The initial temperature is T. Determine the following in terms of N, V, and T:
(a) The initial pressure P_i.
(b) The change ΔU of the internal energy.
(c) The final temperature T_f.
(d) The final pressure P_f.

(2) An Einstein solid consists of $N = 4$ identical and independent simple harmonic oscillators which interact with each other only so that they can slowly exchange energy. The solid contains a total energy $U = q\epsilon$ where $q = 4$ and $\epsilon = hf$ is the fundamental energy quantum of the oscillators. In terms of ϵ and Boltzmann's constant k,

(a) Find the multiplicity of the solid.
(b) Find the entropy of the solid.
(c) Estimate the temperature of the solid.

You are welcome to use a computer or calculator, but the calculation is not hard to do by hand and you can leave the answers in terms of the natural logarithm, where appropriate.

(3) Calculate the internal energy U in Joules for one mole of hydrogen gas at $T = 400$K.

(4) Consider two blocks made of the same steel, with specific heat capacity 0.5 J/g·K and density 10 g/cm^3. One block is a cube 3 cm on a side, and the other is rectangular with dimensions $3 \times 3 \times 9$ cm^3. Initially, the cube is at a temperature of 20°C and the rectangular block is at 100°C. The blocks are brought into thermal contact so that they are completely isolated from the outside world. After the blocks have come to thermal equilibrium,
(a) Find the final temperature of the pair of blocks. *Explain your reasoning.*
(b) Calculate the total change in entropy for the pair of blocks. If you don't calculate out the natural logarithms, then at least indicate the sign of ΔS.

Solutions

(1) (a) From (1.8) $P_i = NkT/V$. (b,c,d) Since there is no heat, the change in energy comes only from the work W done on the gas. From (1.40) $V^\gamma P = \text{constant} = V_i^\gamma P_i$ where $\gamma = (f+2)/f = 5/3$ since $f = 3$ for a monatomic ideal gas. Therefore, also using (1.33),

$$\begin{aligned}\Delta U &= -\int_{V_i}^{V_f} PdV = -V_i^\gamma P_i \int_{V_i}^{V_f} \frac{dV}{V^\gamma} = -\frac{V_i^\gamma P_i}{-\gamma+1}\left[\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right] = \frac{P_i V}{\gamma - 1}\left[2^{\gamma-1} - 1\right] \\ &= \frac{3}{2}NkT\left[2^{2/3} - 1\right] = \frac{3}{2}Nk(T_f - T_i) \text{ so } T_f = 2^{2/3}T \text{ and } P_f = NKT_f/V_f = 2^{5/3}P_i\end{aligned}$$

(2) From (2.9) the multiplicity is $\Omega(N,q) = (q+N-1)!/q!(N-1)!$. We will need

$$\Omega(4,4) = \frac{7!}{4!3!} = \frac{7\cdot 6\cdot 5}{3\cdot 2} = 35, \Omega(4,3) = \frac{6!}{3!3!} = \frac{6\cdot 5\cdot 4}{3\cdot 2} = 20, \Omega(4,5) = \frac{8!}{5!3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2} = 56$$

(a) $\Omega(4,4) = 35$, (b) $S = k\log\Omega = k\log 35$, (c) Approximate $1/T = \partial S/\partial U = (1/\epsilon)\partial S/\partial q$ as we did in class, using the values on either side, that is

$$\frac{1}{T} = \frac{1}{\epsilon}\frac{\partial S}{\partial q} \approx \frac{1}{\epsilon}\frac{k\log 56 - k\log 20}{5-3} = \frac{k}{2\epsilon}\log\frac{14}{5} \qquad \text{so} \qquad T \approx \frac{2\epsilon}{k}\frac{1}{\log(14/5)} = 1.94\frac{\epsilon}{k}$$

(3) From (1.23) $U = fNkT/2 = fnRT/2$ using (1.7). From (1.46) and Figure 1.13, $f = 5$ for hydrogen at 400 K. Therefore, with $R = 8.314$ J/mol·K (from the inside back cover),

$$U = 5 \times 1 \times 8.314 \times 400/2 = 8314 \text{ J}$$

(4) (a) Energy is conserved and can only flow between the blocks, and there are no volume changes or other energy sources, so they equilibrate by heat transfer. Using $Q = C\Delta T = cm\Delta T$ from (1.41) and (1.42), we must have $cm_1(T_1 - T) = cm_2(T - T_2)$ where block #1 is the (cold) cube and #2 is the (hot) rectangle, and T is the final temperature. Therefore

$$T = \frac{m_1T_1 + m_2T_2}{m_1 + m_2} = \frac{T_1 + (m_2/m_1)T_2}{1 + m_2/m_1} = \frac{20 + 3\times 100}{1+3} = 80^\circ\text{C}$$

where the second form makes it clear that only the ratio of masses matters. (It is worth noting that for $m_1 = m_2$, T is just the average of the two temperatures.) (b) From (3.18), and (3.19) and following (3.20), $\Delta S = mc\log(T_f/T_i)$ for each block. Therefore

$$\Delta S = \Delta S_1 + \Delta S_2 = m_1c\log\frac{80+273}{20+273} + m_2c\log\frac{80+273}{100+273} = m_1c\left[\log\frac{353}{293} + 3\log\frac{353}{373}\right]$$

With $m_1 = 81\times 10 = 810$ g, $C = mc = 405$ J/K and $\Delta S = 8.5$ J/K> 0.