PHYS4000 Astrophysics
(Spring 2017)

Final Exam
Friday 10 March 2017

There are **five questions**, equally weighted, and you are to work all of them. You are welcome to use your textbook, notes, or any other resources, other than consulting with another human. If you have questions, please ask the person proctoring the exam.

Please start each problem on a new page in your exam booklet.

Good luck!

(1) Match the image (on the sheet handed out separately) with the following descriptions:

(i)	Neutron star	(iv)	White dwarf	(vii)	H-II Region
(ii)	Most distant	(v)	Young star cluster	(viii)	Old star cluster
(iii)	$> 10^9\ M_\odot$ black hole	(vi)	Microwave image	(ix)	X-Ray image

Obviously, not all the images are used, and some images may be used more than once.

(2) Two stars execute circular orbits about their common center of mass. The masses of the stars are four and twelve solar masses respectively. The orbital period is two years.
(a) What is the separation between the two stars?
(b) What is the orbital speed of the four solar mass star?
(c) For an inclination angle $i = 30^\circ$, what are the maximum and minimum Doppler shifted wavelengths observed for the $H\alpha$ line from the four solar mass star?
(d) What is the farthest distance away these stars can be, if they are to be distinctly resolved by the Hubble Space Telescope. (Assume an angular resolution of 0.1 seconds of arc.)

(3) Assume that the Universe was radiation dominated up until the time that it was cool enough so that blackbody photons could no longer effectively ionize hydrogen. Recall that there were (and are) about 10^9 photons for every proton in the Universe.
(a) Give a rough estimate of the temperature at which this transition took place. Justify your answer, although you don't need to make a detailed calculation.
(b) How old was the Universe when this decoupling took place?

(4) A galaxy with redshift $z = 0.2$ has a total luminosity equal to $10^{10}\ L_\odot$. Find the observed flux on Earth, in either Joules/m^2·sec or ergs/cm^2·sec.

(5) The Kepler mission has detected a distinctly Earth-like extrasolar planet in Lyra. It has a radius $R_p = 1.34\ R_\oplus$ and orbits a star with radius $R_\star = 0.6\ R_\odot$ at an orbital radius close to 0.4 AU. The orbit is nearly circular with an inclination angle close to 90°, and with a 112 day orbital period. Sketch the transit signal as seen by Kepler, including details of the transit time and signal size.

Solutions

(1) (i) F; (ii) K; (iii) K or L; (iv) B or I; (v) H; (vi) C; (vii) J; (viii) E; (ix) I or F

(2) Have $m_1r_1 = m_2r_2$, $m_1 = 4M_\odot$, $m_2 = 12M_\odot$, $F = Gm_1m_2/a^2$, and $a = r_1 + r_2 = 4r_2$.

$$G\frac{m_1m_2}{16r_2^2} = m_2\frac{v_2^2}{r_2} = 4\pi^2 m_2\frac{r_2}{\tau^2} \quad \text{so} \quad r_2 = \left(\frac{GM_\odot}{16\pi^2}\tau^2\right)^{1/3} \quad \text{and} \quad a = \left(\frac{GM_\odot}{\pi^2}\tau^2\right)^{1/3}$$

Also $v_2 = 2\pi r_2/\tau$ and $\Delta\lambda/\lambda = v_2 \sin i/c$ with $\lambda = 656$ nm. Finally $\theta = a/d$ so solve for d with $\theta = 0.1 \times \pi/(180 \times 3600)$. Calculating all this gives

$a = 6.0 \times 10^{11}$ m= 4 AU; $v_1 = 4.5 \times 10^4$ m/sec; $\lambda = 656 \pm 0.05$ nm; $d = 1.2 \times 10^{18}$ m= 40 pc.

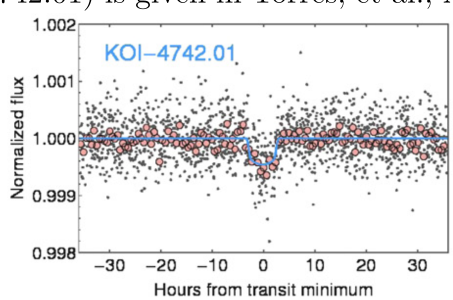
Note that the first part can be done simply using solar masses, years, and AU:

$$G\frac{m_1m_2}{16r_2^2} = 4\pi^2 m_2\frac{r_2}{\tau^2} \quad \text{so} \quad \left(\frac{m_1}{m_\odot}\right)\left(\frac{\tau}{\text{year}}\right)^2 = 16\left(\frac{r_2}{\text{AU}}\right)^2 \quad \text{so} \quad r_2 = 1 \text{ AU}$$

(3) The number of black body photons with energy $h\nu$ is proportional to $\nu^2/(e^{h\nu/kT} - 1)$, so, we want the temperature at which the integral from 13.6 eV to ∞ is 10^{-9} of the total. Some trial and error with MATHEMATICA shows that $T = 5965$ K gives 1.0×10^{-9} for the number of photons above 13.6 eV. (I think in class, we used a number from the book around 3000 K.) Now, using $T = 1.5 \times 10^{10}\ \text{K} \cdot \text{sec}^{1/2}/t^{1/2}$, we find $t = 6.5 \times 10^{12}$ sec= 2.0×10^5 years.

(4) Use $cz = v = H_0 d$ with $H_0 = 72$ km/sec·Mpc to find $v = 59958$ km/sec and $d = 833$ Mpc. The flux is therefore $\phi = L/4\pi d^2 = 4.6 \times 10^{-16}$ J/m^2·sec= 4.6×10^{-13} erg/cm^2·sec.

(5) This describes the planet Kepler-442b. It is probably easiest to simply refer to formulas in the textbook. From (6.6) $\Delta f/f = (R_p/R_\star)^2 = 4.2 \times 10^{-4}$. For the transit duration use (6.7) with $b = 0$ since $i = 90^\circ$ since $\cos i = br_\star/a$. Thus $t_{\text{dur}} = 0.25$ days. Data for this planet (also known as KOI-4742.01) is given in Torres, et al., Ap.J. 800:99(2015)1. The plot



indeed shows a "dip" of size that is about 1/2 of 0.001, and a transit that takes several hours.