PHYS4000 Astrophysics
(Spring 2017)

Midterm Exam #1
Friday 10 March 2017

There are **four questions** and you are to work all of them. You are welcome to use your textbook, notes, or any other resources, other than consulting with another human. Of course, if you have questions, you are encouraged to ask the person proctoring the exam.

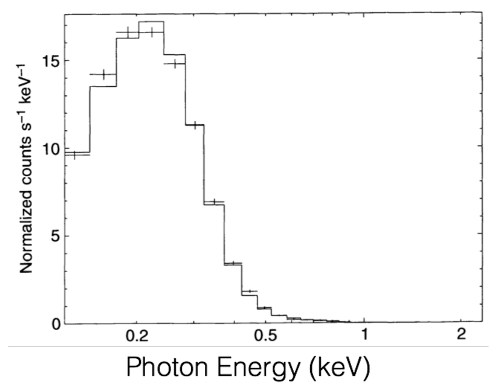
Please start each problem on a new page in your exam booklet.

Good luck!

(1) Two stars orbit each other and spectral data indicates they are each 1 kpc= 3.1×10^{19} m away. With a 1 m diameter aperture telescope, the stars can be resolved in blue light, but they cannot be resolved in red light. Estimate the distance between the two stars.

(2) Star #1 is 256 times brighter than star #2. They are at the same distance from us.
a. If the stars have the same radius, what is the ratio of their surface temperatures?
b. If the stars have the same surface temperatures, what is the ratio of their radii?
c. If both are Main Sequence stars, estimate the ratio of their masses.

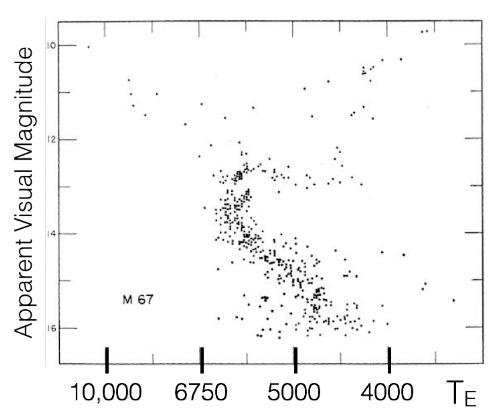
(3) The figure to the right shows the thermal X-ray energy spectrum of an isolated neutron star. (Note that X-ray detectors record the energy of single photons, as opposed to the wavelength of the "light.")



a. Estimate the surface temperature of the "star."

b. Assuming a one solar mass neutron star, estimate the total luminosity and compare to that of the Sun.

(4) The figure to the right shows an HR diagram for the star cluster M67. The vertical axis is apparent visual magnitude, and the horizontal axis is effective surface temperature in Kelvin (K).



a. Where does the Sun fall on this diagram? (You can describe in words or a drawing in your exam booklet.)

b. Use the position of the Sun to estimate the age of this star cluster. Explain your reasoning.

Solutions

(1) Red is 600 nm and blue is 400 nm so take $\lambda = 500$ nm$= 5\times10^{-7}$ m, so $\theta = \lambda/D = 5\times10^{-7}$ is the diffraction limited angle. The stars are separated by $\theta d = 15 \times 10^{12}$ m$=$ 100 AU.

(2) $L_1 = 256L_2 = 2^8L_2$ and $L \sim R^2T^4$, so

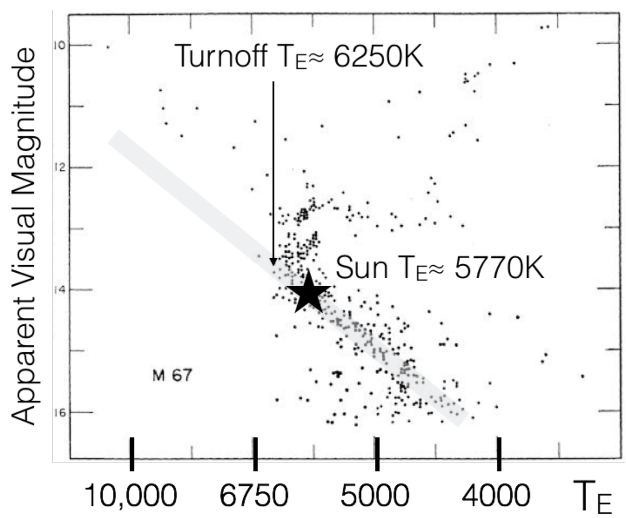
a. $L_1/L_2 = T_1^4/T_2^4 = 2^8$ and $T_1/T_2 = 4$.

b. $L_1/L_2 = R_1^2/R_2^2 = 2^8$ and $R_1/R_2 = 16$.

c. $L \sim M^3$ on the Main Sequence, so $M_2/M_1 \approx (256)^{1/3} \approx 6$.

(3) The blackbody spectrum peaks at $E_{\rm max} = 0.2$ keV. Wien says $T = 0.29\ {\rm cm}\cdot{\rm K}/\lambda_{\rm max}$ and $E = h\nu = hc/\lambda = 2\pi\hbar c/\lambda$ so $\lambda_{\rm max} = 2\pi \times 200\ {\rm MeV}\cdot{\rm fm}/0.2\ {\rm keV} = 2\pi$ nm and $T = 5\times10^5$ K. The radius of a 1 $M_\odot$ neutron star is $R = 11\ {\rm km}\times 1.4^{1/3} = 12$ km, and $L = 4\pi R^2\sigma T^4$ so find $L = 6.4\times10^{31}$ erg/sec$=$ $1.7\times10^{-2}\ L/L_\odot$. Agrees with Nature 379(1996)233. The numbers are in the MATHEMATICA notebook.

(4) The Main Sequence is along the gray bar, drawn on the figure on the right. It passes through the group of cooler stars in M67. The Sun sits on the Main Sequence, with an effective temperature near 5770 K. The turnoff is at a somewhat higher effective temperature, so the age of the cluster is somewhat shorter than the lifetime of the Sun, i.e. 10^{10} years. Indeed, age estimates for M67 vary between 3×10^9 and 5×10^9 years, easy enough to find on Wikipedia.



To do a better job on the lifetime, we need to know how age t and luminosity L scale with effective temperature T and mass M. We have $L \sim T^8$ and $L \sim M^3$ so $M \sim T^{8/3}$. We also know that $t \sim 1/M^2$ so $t \sim 1/T^{16/3}$. Therefore, we estimate the age of the cluster as

$$t({\rm M67}) = 10^{10}\ {\rm yr}\times(5770/6250)^{16/3} = 6.5\times10^9\ {\rm yr}$$