PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #14

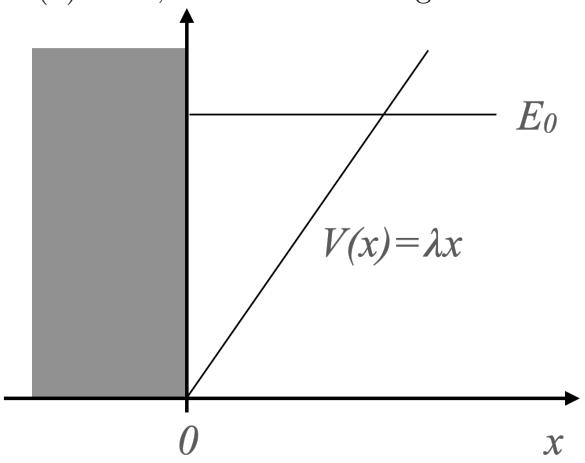
Due at 5pm to the **Grader** on Thursday 22 Apr 2021

(1) The Hamiltonian for a spin-1/2 particle of mass m and charge q in a magnetic field $\mathbf{B}$ is

$$\hat{H} = -\hat{\boldsymbol{\mu}} \cdot \mathbf{B} = -\frac{gq}{2mc}\hat{\mathbf{S}} \cdot \mathbf{B}$$

where g is the gyromagnetic ratio. The magnetic field has components in the z- and y-directions, and is written as $\mathbf{B} = B_0\hat{\mathbf{k}} + B_1\hat{\mathbf{j}}$. Determine the energy eigenvalues exactly. Then, taking $B_1 \ll B_0$, find the eigenvalues through second order in perturbation theory. Compare the two approaches.

(2) A particle of mass m is bound in one dimension $x \geq 0$ by an infinite wall at $x = 0$ and a linear potential energy $V(x) = \lambda x$, as shown in the figure below.



Use the variational principle to estimate the ground state E_0 using the two different trial functions (i) $\psi(x) = xe^{-x/2a}$ and (ii) $\psi(x) = xe^{-x^2/2a^2}$, where a is a variable parameter. (The calculation can be done analytically, but you are welcome to use MATHEMATICA or some other application.) Why does choice (ii) give you a better approximation to the exact answer? (How do you know it is the better approximation?)

Explain how this can be measured by observing the quantized vertical heights to which a ball can bounce off the floor. Calculate the height to which a neutron must bounce in order to reach the ground state. Compare your answer to the experiment described in "Quantum states of neutrons in the Earth's gravitational field," V.V. Nesvizhevsky, et al., Nature 415 (17 January 2002) 297.