# PHYS3701 Introduction to Quantum Mechanics I Spring 2021

## Homework Assignment #13

## Due at 5pm to the **Grader** on Thursday 15 Apr 2021

**(1)** These questions are meant to associate numbers with atomic hydrogen phenomena.

a. The red $n = 3 \rightarrow 2$ Balmer transition has a wavelength $\lambda \approx 656$ nm. Calculate the wavelength difference $\Delta\lambda$ (in nm) between the $3p_{3/2} \rightarrow 2s_{1/2}$ and $3p_{1/2} \rightarrow 2s_{1/2}$ transitions due to the spin-orbit interaction. Comment on how you might measure this splitting.

b. How large an electric field $\mathcal{E}$ is needed so that the Stark splitting in the $n = 2$ level is the same as the correction from relativistic kinetic energy between the $2s$ and $2p$ levels? How easy or difficult is it to achieve an electric field of this magnitude in the laboratory?

c. The Zeeman effect can be calculated with a "weak" or "strong" magnetic field, depending on the size of the energy shift relative to the spin-orbit splitting. Give examples of a weak and a strong field. How easy or difficult is it to achieve such a magnetic field?

**(2)** Fill in details and complete the calculation of the "total angular momentum" eigenvalues and eigenstates, in Section 11.5 of your textbook. In particular

a. Find the matrix representation of the operator $2\hat{\mathbf{L}} \cdot \hat{\mathbf{S}}$ in the basis

$$|1\rangle \equiv |l, m_l, +\mathbf{z}\rangle \qquad \text{and} \qquad |2\rangle \equiv |l, m_l + 1, -\mathbf{z}\rangle$$

b. Show that the eigenvalues of this matrix are $\lambda = l$ and $\lambda = -l - 1$.

c. Show that these correspond to the eigenvalues $j(j+1)\hbar^2$ with $j = l \pm 1/2$ for the operator $\hat{\mathbf{J}}^2$ where $\hat{\mathbf{J}} \equiv \hat{\mathbf{L}} + \hat{\mathbf{S}}$.

d. Show that the expressions (11.93) for the total angular momentum eigenstates

$$|j = l \pm 1/2, m_j\rangle = \sqrt{\frac{l \pm m_j + 1/2}{2l+1}}|l, m_j - 1/2, +\mathbf{z}\rangle \pm \sqrt{\frac{l \mp m_j + 1/2}{2l+1}}|l, m_j + 1/2, -\mathbf{z}\rangle$$

are normalized and give the correct eigenvalues for $\hat{J}_z$.