PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #12

Due at 5pm to the **Grader** on Thursday 8 Apr 2021

(1) Consider a simple harmonic oscillator in one dimension x for a particle of mass m and with a potential energy function $m\omega^2 x^2/2$. Now add a "perturbation" of the form $m\omega_1^2 x^2/2$, where $\omega_1 \ll \omega$. Calculate the energy shifts through second order. Of course, this problem can be solved exactly. Expand the exact energy eigenvalues in powers of ω_1/ω to compare to the perturbation expansion.

(2) This problem asks you to calculate the effect of the finite size of the proton on the energy levels of the hydrogen atom. The proton is a "fuzzy" object with a radius close to 1 fm, but for this calculation let's assume it is a uniformly charged sphere of radius R. First, show that, for the radial coordinate $r \leq R$, the electrostatic potential energy is

$$V(r) = -\frac{3}{2}\frac{e^2}{R^3}\left(R^2 - \frac{1}{3}r^2\right)$$

It is probably easiest to do this by using Gauss' Law to get the electric field inside the sphere, integrating to get the potential, and enforcing continuity at $r = R$.

The difference between this potential energy and the point-like form we used to solve the hydrogen atom, can be treated as a perturbation. Calculate the effect of the energy shift on the $1s$, $2s$, and $2p$ states of the hydrogen atom, that is, the states $|100\rangle$, $|200\rangle$, and $|21m\rangle$. Why do you not need to be concerned about the degeneracy in $n = 2$?

It will be best to use MATHEMATICA or some similar application to carry out the necessary integrations. Argue that, physically, $R \ll a_0$ and use this to simply your expressions. It is easy in MATHEMATICA to use the Series function to expand any expression in some small quantity.

If you write your answers for $E^{(1)}_{1,0}$, $E^{(1)}_{2,0}$, and $E^{(1)}_{2,1}$ as a numerical value times e^2/a_0 times a power of R/a_0, you will find that the $l = 0$ and $l = 2$ states have very different dependences on R/a_0. What is the physical reason for this difference?