PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #11

Due at 5pm to the <u>**Grader**</u> on Thursday 1 Apr 2021

(1) Consider Problem #1 from Homework #10, the assignment from last week. Show that there exists *another* solution that satisfies the same criteria, but with a much deeper well. This is not a physically valid solution only because, in this case, $E = -2.2$ MeV is the energy of the first excited state, and there is no experimental evidence for a more deeply bound ground state in deuterium. Nevertheless, find the energy of this fictional ground state, and make a plot of the wave functions for the ground and first excited states. How do these plots compare to the one you made last week?

(2) Consider a particle of mass m bound by an isotropic harmonic oscillator potential in *two* dimensions, that is

$$V(r) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\omega^2(x^2 + y^2)$$

a. Show that the Hamiltonian $\hat{H}$ naturally splits into the sum of two commuting Hamiltonians $\hat{H}_x$ and $\hat{H}_y$, similar to the way we treated the three dimensional case in class.

b. Show that the energy eigenvalues are $E_n = (n+1)\hbar\omega$ for $n = 0, 1, 2, \ldots$, and express n in terms of the quantum numbers n_x and n_y for $\hat{H}_x$ and $\hat{H}_y$.

c. Discuss the level of energy degeneracy for the lowest few eigenvalues.

d. Define an orbital angular momentum operator in the usual way, namely

$$\hat{L} \equiv \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$$

You can express $\hat{L}$ in terms of the creation and annihilation operators corresponding to $\hat{H}_x$ and $\hat{H}_y$, as in Equations (7.11) and (7.12) in your textbook. Prove explicitly that $[\hat{H}, \hat{L}] = 0$. What symmetry argument can you give that says this is exactly what you should expect?

e. There are two eigenstates $|n_x, n_y\rangle$ with the same energy $E_1 = 2\hbar\omega$. Determine the correct linear combinations of these that are eigenstates of $\hat{L}$. (They have to exist, because $[\hat{H}, \hat{L}] = 0$, right?) You might be able to guess them, but you can also just diagonalize $\hat{L}$ in the $|n_x, n_y\rangle$ space. What are the eigenvalues of $\hat{L}$?