PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #10

Due at 5pm to the **Grader** on Thursday 25 Mar 2021

(1) The deuteron is the simplest atomic nucleus, the bound state of one proton and one neutron. Experiments show that they are in a relative s-state, that is, $\ell = 0$, and that the binding energy, that is, the energy that needs to be added to separate the proton and neutron, is 2.2 MeV. Experiments also show that the force that binds the proton and neutron can be represented by a finite spherical well of radius $a = 1.7$ fm, that is, twice the RMS radii of the proton and neutron. Use this data to find the depth, in MeV, of the finite spherical well. Some computer programming is required to solve the resulting transcendental equations.

Your textbook outlines a graphical approach to estimate the solution, but I think it is easier to transform the equation to solve for $v \equiv V_0/|E|$ using MATHEMATICA or some other tool. I wrote the appropriately rearranged version of (10.60), plotted both sides to estimate the solution, then used a numerical function to determine the intersection.

When you have the well depth, make a plot that combines the shape of the potential well, a line for the energy level, and the (normalized) wave function $u(r)$. This is Figure 10.9 in your textbook.

You might be amused to know that I solved this problem as an undergraduate in 1976. The numerical solution required me to write a FORTRAN program, typed onto punch cards, which were fed into an IBM 360 Mainframe computer.

(2) An electron is in the ground state of tritium, a nucleus with one proton and two neutrons. Tritium is radioactive, and beta decays to the nucleus ^{3}He, with two protons and one neutron. The decay happens, for all purposes, instantaneously. Calculate the (numerical) probability that the electron remains in the ground state of the $^3\text{He}^+$ ion.