PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #8

Due at 5pm to the <u>**Grader**</u> on Thursday 11 Mar 2021

(1) The uncertainties in position Δx and momentum Δp_x for some state $|\psi\rangle$ are determined from the expectation values of x, x^2, p_x, and p_x^2 by

$$(\Delta x)^2 = \left\langle (x - \langle x \rangle)^2 \right\rangle = \langle x^2 \rangle - \langle x \rangle^2 \qquad \text{and} \qquad (\Delta p_x)^2 = \left\langle (p_x - \langle p_x \rangle)^2 \right\rangle = \langle p_x^2 \rangle - \langle p_x \rangle^2$$

a. Determine Δx and Δp_x for the ground state $|0\rangle$ of a particle of mass m in a harmonic oscillator potential energy $V(x) = m\omega^2 x^2/2$. Calculate the expectation values by integrating over x. (The normalized ground state wave function is Equation (7.44) in your textbook, and you are welcome to do the integrals using MATHEMATICA, lookup tables, or whatever other modern convenience you have at your disposal.) Form the product $\Delta x \Delta p_x$ and show that it is the minimum value allowed by the uncertainty relation (3.74).

b. Determine Δx and Δp_x for an arbitrary energy eigenstate $|n\rangle$ using operator methods, that is, writing $\hat{x}$ and $\hat{p}_x$ in terms of $\hat{a}$ and $\hat{a}^\dagger$. Show that your answer for $n = 0$ agrees with what you calculated above for the ground state.

(2) Consider a particle of mass m in a harmonic oscillator potential energy $V(x) = m\omega^2 x^2/2$. The initial state is given in terms of energy eigenstates $|n\rangle$ by

$$|\psi(t = 0)\rangle = a|n\rangle + b|n + 1\rangle$$

where a and b are, in principle, complex constants. That is, a superposition of two adjacent energy eigenstates. The state $|\psi(t = 0)\rangle$ is, of course, properly normalized.

a. Calculate the expectation values $\langle x \rangle$ and $\langle p_x \rangle$, both as a function of time. Comment on the result if either a or b is zero, and the effect of the relative phases of a and b on the result. Feel free to modify the notation if you think that is helpful.

b. For $n = 0$ and $a = b = 1/\sqrt{2}$, your answer should follow, and agree with, Example 7.3 in your textbook. Calculate and plot the probability density $|\langle x|\psi(t)\rangle|^2$ for times $t = 0$, $t = \pi/2\omega$, and $t = \pi/\omega$, as shown in Figure 7.10. (If you know how to do it, write a program that animates this.) Comment on how your result compares to $\langle x \rangle$.