PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #7

Due at 5pm to the **Grader** on Thursday 4 Mar 2021

(1) Prove that $[\hat{x}^n, \hat{p}_x] = i\hbar n\hat{x}^{n-1}$. You can do this using a relationship you proved on Homework #3, namely $\left[\hat{A}\hat{B}, \hat{C}\right] = \hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}$. Then use this result along with the Taylor expansion of $F(x)$ about $x = 0$ to show that

$$[F(\hat{x}), \hat{p}_x] = i\hbar\frac{\partial F}{\partial x}(\hat{x})$$

Finally, show that the Hamiltonian

$$\hat{H} = \frac{1}{2m}\hat{p}_x^2 + V(\hat{x}) \qquad \text{gives} \qquad \frac{d\langle p_x\rangle}{dt} = \left\langle -\frac{dV}{dx}\right\rangle$$

and explain how this is equivalent to Newton's Second Law of Motion.

(2) A particle of mass m sits in a one-dimensional infinite well such that $V(x) = 0$ for $0 \le x \le L$. The particle cannot be found outside the well.

a. Find the energy eigenvalues and their eigenfunctions. Compare your answers to the eigenfunctions (6.106) and eigenvalues (6.110) in the textbook.

b. Suppose that the wave function at time $t = 0$ is given by

$$\psi(x, t = 0) = \left(\frac{1+i}{2}\right)\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L} + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

for $0 \le x \le L$ and zero elsewhere. Prove that this wave function is properly normalized. You can do this by integrating $\psi^*\psi$ over $0 \le x \le L$, but there is an easier way.

c. Find the wave function $\psi(x, t)$, that is, as a function of time.

d. Calculate the expectation value $\langle E\rangle$ of the energy. You can do this by integrating over $0 \le x \le L$, but there is an easier way. Why is $\langle E\rangle$ independent of time?

e. Calculate the expectation value $\langle x\rangle$ of the particle position, as a function of time. This is a little tedious, unless you us something like Mathematica.

f. What is the probability, as a function of time, that a measurement of the energy will yield the value $\hbar^2\pi^2/2mL^2$. Once again, you can do this by integrating over $0 \le x \le L$, but there is an easier way.