PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #6

Due at 5pm to the **Grader** on Thursday 25 Feb 2021

(1) Work through the details of the discussion in Section 4.4 of the textbook and derive the Rabi formula (4.45) for the probability (as a function of time) of observing the "spin-down" state $|-\mathbf{z}\rangle$, when it is initally in the "spin-up" state $|-\mathbf{z}\rangle$, in terms of the strengths B_0 and B_1 of the static and oscillating magnetic fields, and the oscillation frequency ω, namely

$$|\langle -\mathbf{z}|\psi(t)\rangle|^2 = \frac{\omega_1^2/4}{(\omega_0-\omega)^2+\omega_1^2/4}\sin^2\frac{\sqrt{(\omega_0-\omega)^2+\omega_1^2/4}}{2}t$$

where $\omega_0 = gqB_0/2mc$ and $\omega_1 = gqB_1/2mc$. First, follow the text to obtain the matrix equation (4.41). (Be careful of the signs of the exponents.) Then, explain (as in the text) why it is OK to ignore the terms with $\omega + \omega_0$ and rewrite the simplified matrix equation.

Solve this pair of simplified equations to find $c(t)$ and $d(t)$, but not with assuming that $\omega = \omega_0$. You can do this by assuming that $c(t)$ and $d(t)$ have a $e^{i\alpha t}$ time dependence, and then solve algebraically for α. There are two solutions, and you can use the initial conditions to find the right linear combination. The Rabi equation comes from the inner product of the row matrix (0 1), that is the representation of $\langle -\mathbf{z}|$ in the $|\pm\mathbf{z}\rangle$ basis, and the column matrix of $a(t)$ and $b(t)$, that is the representation of $|\psi(t)\rangle$ in the same basis.

Plot the amplitude of the oscillations as in Figure 4.6 in the textbook, that is the maximum value of the transition probability as a function of the frequency ω of the oscillating field. If you write $\omega = x\omega_0$, then you can plot it against x for values near unity. Make the plot for three different values of the ratio $B_1/B_0 = 10^{-2}$, 10^{-3}, and 10^{-4}. These might be typical ratios for a Magnetic Resonance Imaging (MRI) scanner. Does this help you see MRI machines are valuable medical diagnostic tools?

(2) *This is a general question based on generic states and Hamiltonian. Do not confuse this with "spin" or "angular momentum."* In some three-state system with states labeled $|1\rangle$, $|2\rangle$, and $|3\rangle$, suppose that the matrix representation of the Hamiltonian is

$$\begin{pmatrix} E_0 & 0 & A \\ 0 & E_1 & 0 \\ A & 0 & E_0 \end{pmatrix}$$

a. If the initial state $|\psi(0)\rangle = |2\rangle$, find the state $|\psi(t)\rangle$ as a function of time.

b. If the initial state $|\psi(0)\rangle = |3\rangle$, find the state $|\psi(t)\rangle$ as a function of time.

Express your answers as time-dependent linear combinations of $|1\rangle$, $|2\rangle$, and $|3\rangle$. Remember that it is always easiest to find the time evolution of a state with a time-independent Hamiltonian by expressing the state in terms of the energy eigenstates.