# PHYS3701 Introduction to Quantum Mechanics I Spring 2021

## Homework Assignment #5

## Due at 5pm to the **Grader** on Thursday 18 Feb 2021

**(1)** *This problem reiterates what we covered in class on Thursday 12 Feb.* Label the combined states of two spin-1/2 particles as we did in class, and also (5.8) in the textbook, as

$$|1\rangle = |+\mathbf{z}, +\mathbf{z}\rangle \qquad |2\rangle = |+\mathbf{z}, -\mathbf{z}\rangle \qquad |3\rangle = |-\mathbf{z}, +\mathbf{z}\rangle \qquad |4\rangle = |-\mathbf{z}, -\mathbf{z}\rangle$$

Then, for the operator $\hat{\mathbf{S}}^2$ from (5.26a) with (5.10),

$$\hat{\mathbf{S}}^2 = \hat{\mathbf{S}}_1^2 + \hat{\mathbf{S}}_2^2 + 2\hat{S}_{1_z}\hat{S}_{2_z} + \hat{S}_{1_+}\hat{S}_{2_-} + \hat{S}_{1_-}\hat{S}_{2_+}$$

calculate the matrix elements $\langle 1|\hat{\mathbf{S}}^2|1\rangle$, $\langle 1|\hat{\mathbf{S}}^2|2\rangle$, $\langle 2|\hat{\mathbf{S}}^2|2\rangle$, $\langle 2|\hat{\mathbf{S}}^2|3\rangle$, $\langle 3|\hat{\mathbf{S}}^2|3\rangle$, and $\langle 4|\hat{\mathbf{S}}^2|4\rangle$. Use this to complete the full $4 \times 4$ matrix representation of $\hat{\mathbf{S}}^2$ in the $|1\rangle$, $|2\rangle$, $|3\rangle$, $|4\rangle$ basis. Find the eigenvalues and eigenvectors. Finally, show that the eigenvectors are also eigenvectors of $\hat{S}_z = \hat{S}_{1_z} + \hat{S}_{2_z}$ and derive the eigenvalues.

**(2)** *I want to try something different on this problem.* The calculation in (1) above is integral to calculating the "hyperfine splitting" in atomic hydrogen. We mentioned this in class, but the result is derived in detail in Section 5.2 of your textbook. The transition between these states corresponds to electromagnetic waves with frequency $\nu = 1420$ MHz, or a wavelength close to 21 cm. It was first measured in the laboratory (not in the Galaxy, as I said mistakenly in class) by Nafe and Nelson, Phys.Rev. **73** (1948) 718. (This seems to be at odds with Footnote 2 on page 146 of your textbook.) Its discovery in the Galaxy was first observed by Ewen and Purcell, Nature **168** (1951) 356.

I want each of you to find a *different* journal article that reports on a measurement that involves the so-called 21 cm line in atomic hydrogen. Most of these will likely be in papers on astrophysics, but I think you can also find some particularly precise laboratory measurements. Any use of this radiation is fine, including papers that show results on galactic rotation curves based on the Doppler shift.

To make sure you each come up with a unique reference, communicate with each other, either using GroupMe or through me or whatever you'd like.