PHYS3701 Introduction to Quantum Mechanics I Spring 2021

Homework Assignment #4

Due at 5pm to the <u>**Grader**</u> on Thursday 11 Feb 2021

(1) In a notation where angular momentum eigenstates are written as usual in the form $|j,m\rangle$, calculate the following matrix elements. Where appropriate, use the Kroneker δ, that is $\delta_{kl}=1$ if $k=l$ and $\delta_{kl}=0$ if $k\neq l$.

a. $\langle j,m|\hat{\mathbf{J}}^2|j,m\rangle$

b. $\langle j',m'|\hat{\mathbf{J}}^2|j,m\rangle$

c. $\langle j',m'|\hat{J}_z|j,m\rangle$

d. $\langle j',m'|\hat{J}_\pm|j,m\rangle$

e. $\langle \frac{1}{2},\frac{1}{2}|J_+|\frac{1}{2},-\frac{1}{2}\rangle$

f. $\langle \frac{3}{2},\frac{1}{2}|J_+|\frac{1}{2},-\frac{1}{2}\rangle$

g. $\langle \frac{3}{2},\frac{1}{2}|J_+|\frac{3}{2},-\frac{1}{2}\rangle$

(2) We show in class on February 9th (and as derived in Section 3.6 of the textbook) that for a normal vector $\mathbf{n}=\mathbf{i}\cos\phi+\mathbf{j}\sin\phi$ in the xy plane, the operator $\hat{S}_n=\hat{\mathbf{S}}\cdot\mathbf{n}$ has the correct eigenvalues ($\pm\hbar/2$) regardless of the value of ϕ. We also derived eigenstates $|\pm\mathbf{n}\rangle$, given by equations (3.98) and (3.101) in the textbook.

Extend this to three dimensions, using spherical polar coordinates to describe the polarization direction, that is

$$\mathbf{n}=\mathbf{i}\sin\theta\cos\phi+\mathbf{j}\sin\theta\sin\phi+\mathbf{k}\cos\theta$$

Show once again that you get the correct eigenvalues independent of θ and ϕ, and derive the eigenvectors in terms of θ and ϕ. Do this by solving the eigenvalue problem directly, as in Section 3.6.