Phys 3701 25 Mar 2021

Perturbation theory (Non-Degen.)

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle \leftarrow$$

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

$$\Leftrightarrow \hat{H} = \hat{H}_0 + \lambda\hat{H}_1$$

Also: $\hat{H}_0|\phi_n^{(0)}\rangle = E_n^{(0)}|\phi_n^{(0)}\rangle$

$$|\psi_n\rangle = |\phi_n^{(0)}\rangle + \lambda|\underline{\phi_n^{(1)}}\rangle + \lambda^2|\phi_n^{(2)}\rangle + \dots$$

$$E_n = E_n^{(0)} + \lambda\underline{E_n^{(1)}} + \lambda^2\underline{E_n^{(2)}} + \dots$$

$$(\hat{H}_0 + \lambda\hat{H}_1)\left(|\phi_n^{(0)}\rangle + \lambda|\phi_n^{(1)}\rangle + \lambda^2|\phi_n^{(2)}\rangle + \dots\right)$$

$$= \left(E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots\right)$$

$$\left(|\phi_n^{(0)}\rangle + \lambda|\phi_n^{(1)}\rangle + \lambda^2|\phi_n^{(2)}\rangle + \dots\right)$$

λ^0: $\hat{H}_0|\phi_n^{(0)}\rangle = E_n^{(0)}|\phi_n^{(0)}\rangle$ <u>SOLVED!</u>

$$\boxed{\begin{aligned}
\lambda^1:\ & \hat{H}_0|\phi_n^{(1)}\rangle + \hat{H}_1|\phi_n^{(0)}\rangle \\
& = E_n^{(0)}|\phi_n^{(1)}\rangle + E_n^{(1)}|\phi_n^{(0)}\rangle \qquad * \\
\lambda^2:\ & \hat{H}_0|\phi_n^{(2)}\rangle + \hat{H}_1|\phi_n^{(1)}\rangle \qquad ** \\
& = E_n^{(0)}|\phi_n^{(2)}\rangle + E_n^{(1)}|\phi_n^{(1)}\rangle + E_n^{(2)}|\phi_n^{(0)}\rangle
\end{aligned}}$$

Do $\langle\phi_n^{(0)}|$ on λ^1 Equation:

$$\cancel{\langle E_n^{(0)}|\hat{H}_0|\phi_n^{(1)}\rangle} + \langle\phi_n^{(0)}|\hat{H}_1|\phi_n^{(0)}\rangle$$

$$= \cancel{E_n^{(0)}\langle\phi_n^{(0)}|\phi_n^{(1)}\rangle} + E_n^{(1)}\underbrace{\langle\phi_n^{(0)}|\phi_n^{(0)}\rangle}_{=1}$$

$$E_n^{(0)}\langle\phi_n^{(0)}|\phi_n^{(1)}\rangle$$

$$\Rightarrow E_n^{(1)} = \langle\phi_n^{(0)}|\hat{H}_1|\phi_n^{(0)}\rangle \qquad (1)$$

Now Do $\langle \phi_k^{(0)}|$ on λ^1 equ for $k \neq n$:

$$\langle \phi_k^{(0)} | \hat{H}_0 | \phi_n^{(1)} \rangle + \langle \phi_k^{(0)} | \hat{H}_1 | \phi_n^{(0)} \rangle$$

$$= E_n^{(0)} \langle \phi_k^{(0)} | \phi_n^{(1)} \rangle + E_n^{(1)} \underbrace{\langle \phi_k^{(0)} | \phi_n^{(0)} \rangle}_{=0}$$

$$(E_k^{(0)} - E_n^{(0)}) \langle \phi_k^{(0)} | \phi_n^{(1)} \rangle + \langle \phi_k^{(0)} | \hat{H}_1 | \phi_n^{(0)} \rangle = 0$$

$$\Rightarrow \langle \phi_k^{(0)} | \phi_n^{(1)} \rangle = \frac{\langle \phi_k^{(0)} | \hat{H}_1 | \phi_n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}}$$

$$|\phi_n^{(1)}\rangle = \sum_k |\phi_k^{(0)}\rangle \langle \phi_k^{(0)} | \phi_n^{(1)} \rangle$$

$$= |\phi_n^{(0)}\rangle \langle \phi_n^{(0)} | \phi_n^{(1)} \rangle + \sum_{k \neq n} |\phi_k^{(0)}\rangle \langle \phi_k^{(0)} | \phi_n^{(1)} \rangle$$

Easy to show $\langle \phi_n^{(0)} | \phi_n^{(1)} \rangle = ia$

$a =$ Real #

$$|\psi_n\rangle = (1+ia)|\phi_n^{(0)}\rangle + \sum_{k\neq n} |\phi_k^{(0)}\rangle \underline{\langle \phi_k^{(0)} | \phi_n^{(1)}\rangle}$$

$$e^{ia}|\phi_n^{(0)}\rangle \Rightarrow a=0$$

i.e. $$|\phi_n^{(1)}\rangle = \sum_{k\neq n} |\phi_k^{(0)}\rangle \frac{\langle \phi_k^{(0)} | \hat{H}_1 | \phi_n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}} \quad (2)$$

Do $\langle \phi_n^{(0)}|$ on λ^2 equation:

$$\Leftrightarrow E_n^{(2)} = \sum_{k\neq n} \frac{|\langle \phi_k^{(0)} | \hat{H}_1 | \phi_n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}} \quad (3)$$