Phys 3701 18 Feb

Review: Time Evolution

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$$
$$i\hbar\frac{d}{dt}\hat{U}(t) = \hat{H}\,\hat{U}(t)$$

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}\,|\psi(t)\rangle$$

If $\hat{H}$ is independent of time

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar} = \hat{1} + \left(-\frac{i}{\hbar}\right)\hat{H}t + \frac{1}{2!}\left(-\frac{i}{\hbar}\right)^2\hat{H}^2t^2 + \cdots$$

Eigenstates $\hat{H}|E\rangle = E|E\rangle$

For $|\psi(0)\rangle = |E\rangle$, then

$$|\psi(t)\rangle = e^{-iEt/\hbar}|E\rangle$$ "Stationary States"

$$\hat{U}(t)|E\rangle = e^{-i\hat{H}t/\hbar}|E\rangle$$
$$= \left[\hat{1} + \left(-\frac{i}{\hbar}\right)t\hat{H} + \frac{1}{2!}\left(-\frac{i}{\hbar}\right)^2t^2\hat{H}^2 + \frac{1}{3!}\left(-\frac{i}{\hbar}\right)^2 t^3 \hat{H}^3 + \cdots\right]$$

$\hat{H}$ $|E\rangle$

$$= \left[1 + \left(-\frac{i}{\hbar}\right) t E + \frac{1}{2!}\left(-\frac{i}{\hbar}\right)^2 t^2 E^2 + \cdots \right]$$

$$\xleftrightarrow{\qquad E \qquad} \qquad |E\rangle$$

$$= e^{-iEt/\hbar} \, |E\rangle$$

Expectation Values

Observable A, $\langle A \rangle = \langle \psi(t) | \hat{A} | \psi(t) \rangle$

$$\frac{d}{dt}\langle A \rangle = \frac{i}{\hbar} \langle \psi(t) | \underbrace{[\hat{H}, \hat{A}]}_{=0} | \psi(t) \rangle + \cancel{\langle \psi(t) | \frac{\partial \hat{A}}{\partial t} | \psi(t) \rangle}$$

usually zero.

$= 0$ "A is conserved"

e.g. "Energy is conserved if the Hamiltonian is time independent"

aka "A is a constant of the motion"

<u>Example</u>: Electron Spin-1/2 in $\underset{\sim}{\vec{B} = B_0 \hat{k}}$

$$\text{Energy} = -\vec{\mu}\cdot\vec{B} = -\mu_z B_0$$

$$\Rightarrow \hat{H} = \underline{\underline{\omega_0}}\,\hat{S}_z \qquad \omega_0 = geB_0/2mc$$

$$\hat{U}(t) = e^{-i\omega_0 \hat{S}_z t/\hbar} = e^{-i\hat{S}_z(\omega_0 t)/\hbar}$$

$$= \hat{R}(\phi\hat{k}) \qquad \phi = \omega_0 t$$

Suppose $|\psi(0)\rangle = \underline{\underline{|+\hat{x}\rangle}}$

$$\left|\langle +\hat{x}|\psi(t)\rangle\right|^2 = \underset{\sim}{\cos^2\frac{\omega_0 t}{2}} \qquad (4.25)$$

"Precession w/ period $T = 2\pi/\omega_0$"

$t=0 \Rightarrow \underline{1}$ $\qquad$ $t=\frac{1}{2}T \Rightarrow 0$ $\qquad$ $t=\frac{1}{4}T \Rightarrow \frac{1}{2}$

Today: More Examples

First, continue w/ Tuesday example...

$$|\psi(t)\rangle = e^{-i\omega_0 t/2}\left[\frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{1}{\sqrt{2}}e^{i\omega_0 t}|-\hat{z}\rangle\right]$$

$$|\langle +\hat{z}|\psi(t)\rangle|^2 = \frac{1}{2} = |\langle -\hat{z}|\psi(t)\rangle|^2$$

$$\langle S_x\rangle = \frac{\hbar}{2}\cos(\omega_0 t) \qquad \langle S_y\rangle = \frac{\hbar}{2}\sin(\omega_0 t)$$

Magnetic Moment of Muon

See links on course web site.

Also: muon-g-2.fnal.gov

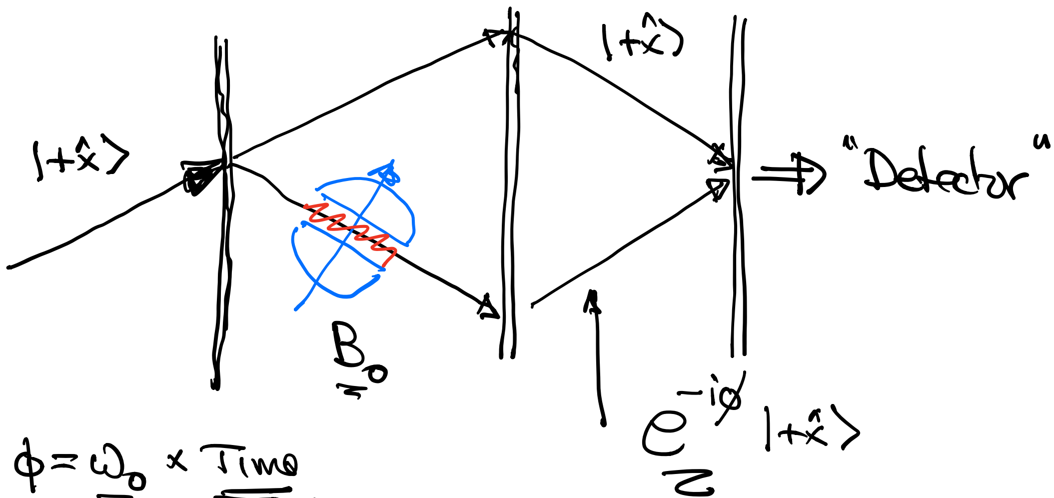
2π Rotations of Spin-1/2 Particle

$$R(\phi\hat{k}) = e^{-i\hat{S}_z\phi/\hbar}$$

$$\Rightarrow R(2\pi)|+\hat{z}\rangle = e^{\overbrace{-i(\hbar/2)2\pi/\hbar}^{-1}}|+\hat{z}\rangle$$

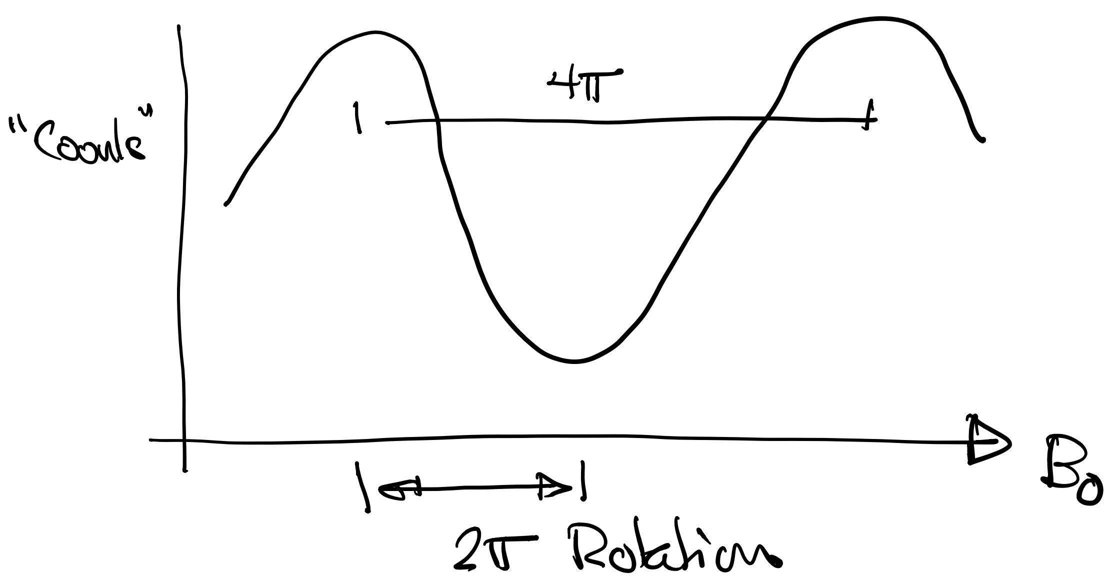
"Neutron Interferometer" Fig. 4.4

Werner, et. al. PRL 35 (1975) 1053



$\phi = \underset{\sim}{\omega}_0 \times \underline{\underline{\text{Time}}}$

$| 1 + e^{-i\phi} |^2$



Magnetic Resonance

$$\hat{H} = -\hat{\vec{\mu}} \cdot \vec{B}(t)$$ piece of $\hat{S}_z$
" " $\hat{S}_x$

$$\vec{B}(t) = B_0 \hat{k} + B_1 \cos(\omega t)\, \hat{i}$$

$$= \frac{1}{2}\left[B_1 \cos(\omega t)\, \hat{i} + B_1 \sin(\omega t)\, \hat{j}\right]$$

$$+ \frac{1}{2}\left[B_1 \cos(\omega t)\, \hat{i} - B_1 \sin(\omega t)\, \hat{j}\right]$$

$|\langle +\hat{z} | \psi(t)\rangle|^2$ for $|\psi(0)\rangle = |+\hat{z}\rangle$

Have to solve $i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Solve as 2×2 matrix problem in $|\pm\hat{z}\rangle$ basis

$$|\psi(t)\rangle = \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} \qquad |\psi(0)\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |+\hat{z}\rangle$$

i.e. want $|a(t)|^2 = |\langle +\hat{z}|\psi(t)\rangle|^2$

$$\Leftrightarrow \quad i\hbar \begin{bmatrix} \dot{a}(t) \\ \dot{b}(t) \end{bmatrix} = \frac{\hbar}{2} \begin{bmatrix} \omega_0 & \omega_1 \cos(\omega t) \\ \omega_1 \cos(\omega t) & -\omega_0 \end{bmatrix} \begin{bmatrix} a(t) \\ b(t) \end{bmatrix}$$

$$\frac{\omega_0}{\omega_1} = \frac{B_0}{B_1}$$

then solve it!!

Eq. (4.45)