Name: ______________________________

PHYS3701 Intro QM I   S24   Quiz #9   11 Apr 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Prove that $\vec{L}\times\vec{L} = i\hbar\vec{L}$ where $\vec{L}$ is the orbital angular momentum operator. You will get full credit if you prove this is true component by component. For 50% extra credit, prove it using (with the usual summation convention) $(\vec{A}\times\vec{B})_i = \epsilon_{ijk}A_jB_k$ and writing the components of $\vec{L}$ as $L_i = \epsilon_{ijk}r_jp_k$, of course being careful to choose summation indices that do not clash.

PHYS3701 Intro QM I   Quiz #9   Solution

Prove that $\vec{L}\times\vec{L} = i\hbar\vec{L}$ where $\vec{L}$ is the orbital angular momentum operator. You will get full credit if you prove this is true component by component. For 50% extra credit, prove it using (with the usual summation convention) $(\vec{A}\times\vec{B})_i = \epsilon_{ijk}A_jB_k$ and writing the components of $\vec{L}$ as $L_i = \epsilon_{ijk}r_jp_k$, of course being careful to choose summation indices that do not clash.

This is very simple if we just look at it by individual components. For example, with the z-component

$$\left(\vec{L}\times\vec{L}\right)_z = L_xL_y - L_yL_x = [L_x, L_y] = i\hbar L_z$$

and similarly for the x- and y-components.

It is not hard to do this with the summation convention for all three components at once.

$$\begin{aligned}
\left(\vec{L}\times\vec{L}\right)_i &= \epsilon_{ijk}L_jL_k \\
&= \epsilon_{ijk}\epsilon_{jmn}r_mp_n\epsilon_{klq}r_lp_q \\
&= \epsilon_{kij}\epsilon_{klq}\epsilon_{jmn}r_mp_nr_lp_q \\
&= (\delta_{il}\delta_{jq} - \delta_{iq}\delta_{jl})\epsilon_{jmn}r_mp_nr_lp_q \\
&= \epsilon_{jmn}r_mp_nr_ip_j - \epsilon_{jmn}r_mp_nr_jp_i \\
&= \epsilon_{jmn}r_mp_n(r_ip_j - r_jp_i) \\
&= \epsilon_{jmn}r_mp_n[r_i, p_j] \\
&= \epsilon_{jmn}r_mp_n\, i\hbar\delta_{ij} \\
&= i\hbar\epsilon_{imn}r_mp_n \\
&= i\hbar L_i
\end{aligned}$$