Name: _____

PHYS3701 Intro QM I   S24   Quiz #8   28 Mar 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Two spin-1/2 particles are emitted from the state $|j, m\rangle = |1, 1\rangle$ in the z-basis, that is a linear combination of the four states $|\pm\hat{\mathbf{z}}\rangle \otimes |\pm\hat{\mathbf{z}}\rangle$. What is the probability that an observer measures particle #1 to be in the $|+\hat{\mathbf{x}}\rangle$ state, while particle #2 is observed to be in the $|-\hat{\mathbf{x}}\rangle$ state? Don't just give the answer; show how to calculate it.

PHYS3701 Intro QM I   Quiz #8   Solution

Two spin-1/2 particles are emitted from the state $|j, m\rangle = |1, 1\rangle$ in the z-basis, that is a linear combination of the four states $|\pm\hat{\mathbf{z}}\rangle \otimes |\pm\hat{\mathbf{z}}\rangle$. What is the probability that an observer measures particle #1 to be in the $|+\hat{\mathbf{x}}\rangle$ state, while particle #2 is observed to be in the $|-\hat{\mathbf{x}}\rangle$ state? Don't just give the answer; show how to calculate it.

As we showed in class, $|1, 1\rangle = |+\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{z}}\rangle$. The observed state is

$$
\begin{aligned}
|+\hat{\mathbf{x}}\rangle \otimes |-\hat{\mathbf{x}}\rangle &= \left(\frac{1}{\sqrt{2}}|+\hat{\mathbf{z}}\rangle + \frac{1}{\sqrt{2}}|-\hat{\mathbf{z}}\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|+\hat{\mathbf{z}}\rangle - \frac{1}{\sqrt{2}}|-\hat{\mathbf{z}}\rangle\right) \\
&= \frac{1}{2}\left(|+\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{z}}\rangle - |+\hat{\mathbf{z}}\rangle \otimes |-\hat{\mathbf{z}}\rangle + |-\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{z}}\rangle - |-\hat{\mathbf{z}}\rangle \otimes |-\hat{\mathbf{z}}\rangle\right)
\end{aligned}
$$

Therefore the probability of observing $|+\hat{\mathbf{x}}\rangle \otimes |-\hat{\mathbf{x}}\rangle$ is

$$
\left|\left(\langle+\hat{\mathbf{x}}| \otimes \langle-\hat{\mathbf{x}}|\right)\left(|+\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{z}}\rangle\right)\right|^2 = \left|\frac{1}{2}(1 - 0 + 0 - 0)\right|^2 = \frac{1}{4}
$$