Name: ____________________

PHYS3701 Intro QM I S24 Quiz #7 21 Mar 2024

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Two spin-1/2 particles are emitted from a spin-zero state. What is the probability that an observer measures particle #1 to be in the $|+\hat{\mathbf{z}}\rangle$ state, while particle #2 is observed to be in the $|+\hat{\mathbf{x}}\rangle$ state? *You are welcome to say what the answer is without a calculation, but it has to be accompanied by a valid explanation. Doing the calculation is perfectly fine, of course.*

PHYS3701 Intro QM I Quiz #7 Solution

Two spin-1/2 particles are emitted from a spin-zero state. What is the probability that an observer measures particle #1 to be in the $|+\hat{\mathbf{z}}\rangle$ state, while particle #2 is observed to be in the $|+\hat{\mathbf{x}}\rangle$ state? *You are welcome to say what the answer is without a calculation, but it has to be accompanied by a valid explanation. Doing the calculation is perfectly fine, of course.*

The answer has to be 1/4 because we know particle #2 is in the $|-\hat{\mathbf{z}}\rangle$ state, which is equal parts $|+\hat{\mathbf{x}}\rangle$ and $|-\hat{\mathbf{x}}\rangle$, and the probability of measuring particle #1 to be in the $|+\hat{\mathbf{z}}\rangle$ state is 1/2. However, it's easy enough to work out the calculation. Express the initial state in the $|\pm\hat{\mathbf{x}}\rangle$ basis because by now, we know by heart how to write $|\pm\hat{\mathbf{x}}\rangle$ in terms of $|\pm\hat{\mathbf{z}}\rangle$. The two-particle state is

$$\begin{aligned}
|\alpha\rangle &= \frac{1}{\sqrt{2}}\left[|+\hat{\mathbf{x}}\rangle \otimes |-\hat{\mathbf{x}}\rangle - |-\hat{\mathbf{x}}\rangle \otimes |+\hat{\mathbf{x}}\rangle\right] \\
&= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|+\hat{\mathbf{z}}\rangle + |-\hat{\mathbf{z}}\rangle\right) \otimes |-\hat{\mathbf{x}}\rangle - \frac{1}{\sqrt{2}}\left(|+\hat{\mathbf{z}}\rangle - |-\hat{\mathbf{z}}\rangle\right) \otimes |+\hat{\mathbf{x}}\rangle\right] \\
&= \frac{1}{2}\left[|+\hat{\mathbf{z}}\rangle \otimes |-\hat{\mathbf{x}}\rangle + |-\hat{\mathbf{z}}\rangle \otimes |-\hat{\mathbf{x}}\rangle - |+\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{x}}\rangle + |-\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{x}}\rangle\right]
\end{aligned}$$

Therefore the probability of observing the state $|+\hat{\mathbf{z}}\rangle \otimes |+\hat{\mathbf{x}}\rangle$ is

$$\left|\left(\langle+\hat{\mathbf{z}}| \otimes \langle+\hat{\mathbf{x}}|\right)|\alpha\rangle\right|^2 = \left|\frac{1}{2}[0+0-1+0]\right|^2 = \frac{1}{4}$$