Name: ______________________________

# PHYS3701 Intro QM I S24 Quiz #6 14 Mar 2024

*You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.*

**Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.**

Calculate the result of rotating the state $|+\hat{\mathbf{x}}\rangle$ by $90^\circ$ about the $z$-axis, that is $\mathcal{D}(\hat{z}, \pi/2)\,|+\hat{\mathbf{x}}\rangle$, and show that you get the answer you expect. You are welcome to work this out in either operator language or using the matrix representation in the $|\pm\hat{\mathbf{z}}\rangle$ basis.

# PHYS3701 Intro QM I Quiz #6 Solution

Calculate the result of rotating the state $|+\hat{\mathbf{x}}\rangle$ by 90° about the $z$-axis, that is $\mathcal{D}(\hat{z},\pi/2)\,|+\hat{\mathbf{x}}\rangle$, and show that you get the answer you expect. You are welcome to work this out in either operator language or using the matrix representation in the $|\pm\hat{\mathbf{z}}\rangle$ basis.

You expect, of course, to get $|+\hat{\mathbf{y}}\rangle$, perhaps with a phase factor in front. In operator language

$$
\begin{aligned}
\mathcal{D}(\hat{z},\pi/2)\,|+\hat{\mathbf{x}}\rangle &= e^{-i\pi S_z/2\hbar}\frac{1}{\sqrt{2}}\left(|+\hat{\mathbf{z}}\rangle + |-\hat{\mathbf{z}}\rangle\right) = \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}\,|+\hat{\mathbf{z}}\rangle + e^{+i\pi/4}\,|-\hat{\mathbf{z}}\rangle\right)\\
&= e^{-i\pi/4}\frac{1}{\sqrt{2}}\left(|+\hat{\mathbf{z}}\rangle + e^{+i\pi/2}\,|-\hat{\mathbf{z}}\rangle\right) = e^{-i\pi/4}\frac{1}{\sqrt{2}}\left(|+\hat{\mathbf{z}}\rangle + i\,|-\hat{\mathbf{z}}\rangle\right) = e^{-i\pi/4}\,|+\hat{\mathbf{y}}\rangle
\end{aligned}
$$

In the $|\pm\hat{\mathbf{z}}\rangle$ representation, the rotation operator becomes

$$
\underline{\underline{\mathcal{D}}}(\hat{z},\pi/2) = \underline{\underline{1}}\cos\frac{\pi}{4} - i\underline{\underline{\sigma}}_z\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1-i & 0 \\ 0 & 1+i \end{bmatrix}
$$

so $\mathcal{D}(\hat{z},\pi/2)\,|+\hat{\mathbf{x}}\rangle$ becomes

$$
\begin{aligned}
\frac{1}{\sqrt{2}}\begin{bmatrix} 1-i & 0 \\ 0 & 1+i \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} &= \frac{1}{2}\begin{bmatrix} 1-i \\ 1+i \end{bmatrix} = \frac{1-i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ (1+i)/(1-i) \end{bmatrix}\\
&= \frac{1-i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ (1+i)^2/2 \end{bmatrix}\\
&= \frac{1-i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = e^{-i\pi/4}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}
\end{aligned}
$$

which is the same result as above.